

PART-A

[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.4 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. [4 × 3 = 12]

- Q.1 If $\frac{2}{\log_b x} = \frac{1}{\log_a x} + \frac{1}{\log_c x}$, where a, b, c, x, belong to $(1, \infty)$, then
- (A) $2b = a + c$ (B) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ (C) $b^2 = ac$ (D) $b^2 = acx$
- Q.2 If $x^2 + y^2 = 14xy$ and $2 \log(k(x+y)) = (\log x + \log y)$, then the value of k is
- (A) $\frac{1}{16}$ (B) $\frac{1}{4}$ (C) $2 \log 2$ (D) $\frac{\log 14}{2}$
- Q.3 Given that $\log_{10}(4252) = 3.6286$ then $\text{antilog}_{10}(0.6286)$ is
- (A) 4.252 (B) 2.52 (C) 3.62 (D) 6.286
- Q.4 If $13^{\log_{10} x} = 338 - x^{\log_{10} 13}$, then the value of $(x+2)$ is equal to
- (A) 13 (B) 15 (C) 17 (D) 102

Q.5 to Q.8 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. [4 × 4 = 16]

- Q.5 If $\log_a 4 = c$, $\log_b a = -1$ and $\log_{\frac{1}{2}} b = -1$ then $(4a^2 + b^2 + c^2)$ equals
- (A) 7 (B) 8 (C) 9 (D) 10
- Q.6 Let $a = \frac{\log_{27} 8}{\log_3 2}$, $b = \left(\frac{1}{2^{\log_2 5}}\right) \left(\frac{1}{5^{\log_5(0.1)}}\right)$ and $c = \frac{\log_4 27}{\log_4 3}$, then the value of $(a + b \div c)$, is
- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{2}{3}$
- Q.7 If $7 \log_p \left(\frac{16}{15}\right) + 5 \log_p \left(\frac{25}{24}\right) + 3 \log_p \left(\frac{81}{80}\right) = 8$, then p^{16} equals
- (A) 16 (B) 1 (C) 2 (D) 4
- Q.8 If $\log_{30}(3) = \alpha$ and $\log_{30}(5) = \beta$, then $\log_{30}(8)$ is equal to
- (A) $3(1 + \alpha - \beta)$ (B) $3(1 + \alpha + \beta)$ (C) $3(\alpha + \beta)$ (D) $3(1 - \alpha - \beta)$

[MULTIPLE CORRECT CHOICE TYPE]

Q.9 to Q.11 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. [3×4=12]

- Q.9 The equation $|x - 10| \log_2(x - 3) = 2(x - 10)$ has
- (A) no prime solution (B) only one natural solution
(C) two rational solutions (D) no composite solution

- Q.10 Which two of the following equations have the same solution
- (A) $x^{\log_{\sqrt{x}}(x-2)} = 9$ (B) $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$
 (C) $\log_4(x + 12) \cdot \log_x 2 = 1$ (D) $\log_3\left(1 + \log_3(2^x - 7)\right) = 1$
- Q.11 Which of the following statement(s) is/are correct?
- (A) $\log_{10} x^2 = 2\log_{10} x$
 (B) The value of $\frac{3^{\log_5 4}}{4^{\log_5 3}}$ is equal to 1.
 (C) $\log_2 5 + \log_5 2 > 2$
 (D) Number of positive integers that logarithm of whose reciprocals to the base 10 has the characteristic (-2) is 90.

PART-B

[MATRIX TYPE]

Q.1 has **four** statements (A, B, C, D) given in **Column-I** and **five** statements (P, Q, R, S, T) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.
[3+3+3+3=12]

Q.1 **Column-I** contains logarithmic equations and entries in **column-II** describes qualitatively the nature of their solution. (Take base of the logarithm as 10 where not mentioned.)

Column-I	Column-II
(A) The equation $x^{\log_3 x} = 9$ has	(P) Only integral solution(s)
(B) The equation $\frac{\log(35 - x^3)}{\log(5 - x)} = 3$, has	(Q) only prime solutions
(C) The equation $9^{\log_{1/3}(x+1)} = 5^{\log_{1/5}(2x^2+1)}$, has	(R) only even integral solution
(D) The equation $4^{(x^2+2)} - (9)2^{(x^2+2)} + 8 = 0$, has	(S) only irrational solutions
	(T) only rational solution(s)

PART-C

[INTEGER TYPE]

Q.1 to Q.3 are "Integer Type" questions. (The answer to each of the questions are upto **4 digits**) **[3×5=15]**

- Q.1 Let $x = 5^{(\log_5 2 + \log_5 3)}$.
 If 'd' denotes the number of digits before decimal in x^{30} and 'c' denotes the number of naughts after decimal before a significant digit starts in x^{-20} , then find the value of $(d - c)$.
 [Take $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$.]
- Q.2 Let $p = \frac{(\log_2 2000)(\log_5 2000) - 4(\log_5 2000)}{\log_2 2000}$. Find $p \in \mathbb{N}$.
- Q.3 If $x = \alpha$ satisfies the equation $\log_6(2^{x+3}) - \log_6(3^x - 2) = x$, then find the value of 3^α .

[STRAIGHT OBJECTIVE TYPE]

[4 × 3 = 12]

- Q.1 The value of $\log_2 \left((\log_{81} 3)^{\log_3 81} \right)$ is equal to
 (A) -8 (B) $-4 \log_2 3$ (C) 8 (D) $-4 \log_3 2$
- Q.2 The real x and y satisfy simultaneously $\log_8 x + \log_4 y^2 = 5$ and $\log_8 y + \log_4 x^2 = 7$ then the value of xy is equal to
 (A) 2^9 (B) 2^{12} (C) 2^{18} (D) 2^{24}
- Q.3 Number of digits in $4^{16} \cdot 5^{25}$ is (use $\log_{10} 2 = 0.3010$)
 (A) 27 (B) 28 (C) 29 (D) 30
- Q.4 Number of real x satisfying the equation $|x - 2| + |x - 3| = |x - 1|$ is
 (A) 1 (B) 2 (C) 3 (D) more than 3

[MULTIPLE OBJECTIVE TYPE]

[1 × 4 = 4]

- Q.5 The expression, $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\dots \sqrt[p]{p}}}}}_{n \text{ radical sign}}$ where $p \geq 2$, $p \in \mathbb{N}$, when simplified is
 (A) independent of p, but dependent on n (B) independent of n, but dependent on p
 (C) dependent on both p & n (D) negative.

[MATCH THE COLUMN]

[3+3+3+3=12]

- | Column-I | Column-II |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
| (A) If $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ then (2x) equals | (P) 1 |
| (B) The number of solutions of $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$ is | (Q) 2 |
| (C) The number of values of x such that the middle term of $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2} \right)$ is the average of the other two is | (R) 3 |
| (D) If α, β are the roots of the equation $x^2 - \left(3 + 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}} \right) x - 2 \left(3^{\log_3 2} - 2^{\log_2 3} \right) = 0$ then $2(\alpha + \beta) - \alpha\beta$ equals | (S) 4 |

[SUBJECTIVE]

- Q.7 The circumference of a circle circumscribing an equilateral triangle is 24π units. Find
 (a) the area of the circle inscribed in the equilateral triangle.
 (b) area of the equilateral triangle inscribed in the inner circle. [3+3]
- Q.8 If $0 < x < \frac{\pi}{4}$ and $\cos x + \sin x = \frac{5}{4}$, find the numerical values of $\cos x - \sin x$. [5]

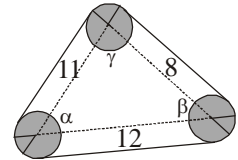
[STRAIGHT OBJECTIVE TYPE]

[8 × 3 = 24]

Q.1 Given $\log_{10} 2 = a$ and $\log_{10} 3 = b$. If $3^{x+2} = 45$. The value of x in terms of a and b is

- (A) $\frac{a-1}{b}$ (B) $\frac{1-a}{b}$ (C) $\frac{1+a}{b}$ (D) $\frac{b}{1-a}$

Q.2 Circles of radius 2 are centered at the vertices of a triangle with sides lengths 8, 11 and 12. Find the length of a belt that fits tightly around those three circles as shown in the figure.



- (A) $31 + 4\pi$ (B) $31 + 6\pi$
(C) $31 + \pi$ (D) $31 + 2\pi$

Q.3 If $\log_9 x + \log_4 y = \frac{7}{2}$ and $\log_9 x - \log_8 y = -\frac{3}{2}$, then $x + y$ equals

- (A) 35 (B) 41 (C) 67 (D) 73

Q.4 Let $a > 1$ be a fixed real number. If S is the set of real number x that are solutions to the equation

$$a^{2\log_2 x} = 5 + 4x^{\log_2 a}, \text{ then}$$

- (A) S contains exactly one real number
(B) S contains more than two, but finitely many, real numbers
(C) S contains exactly two real numbers
(D) S contains infinitely many real numbers

Q.5 The solution set of the equation $\log_{10} (3x^2 + 12x + 19) - \log_{10} (3x + 4) = 1$ is

- (A) a null set (B) a singleton
(C) a set consisting of exactly two elements (D) a set consisting of more than two elements

Q.6 Suppose n be an integer greater than 1, let $a_n = \frac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and

$c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $(b - c)$ equals

- (A) $\frac{1}{1001}$ (B) $\frac{1}{1002}$ (C) -1 (D) -2

Q.7 The value of $\log_2 \left(\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}} \right)$ is equal to

- (A) 1 (B) 0 (C) $1/2$ (D) $\log_2 3$

Q.8 Let $\alpha = \sqrt{19-8\sqrt{3}} + \sqrt{7+4\sqrt{3}}$ and $\beta = \sqrt{83-18\sqrt{2}} - \sqrt{6-4\sqrt{2}}$, then $\log_2 \left(\frac{\alpha}{\beta} \right)$ lies in the interval

- (A) $(-2, -1)$ (B) $\left(\frac{-1}{2}, 0 \right)$ (C) $(0, 1)$ (D) $\left(-1, \frac{-1}{2} \right)$

[SUBJECTIVE]

Q.9 A rail road curve is to be laid out on a circle. If the track is to change direction by 28° in a distance of 44 meters. Find the radius of the curve (in metre). (use $\pi = 22/7$) [4]

Q.10 If $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$, then find the value of $8 \operatorname{cosec}^6 \alpha + 27 \sec^6 \alpha$. [6]