Periodic Functions

(1) $f(x) = \cos\frac{2x}{3} - \sin\frac{4x}{5}$ (15π) (2) $f(x) = \cos(\sin x)$ (π)

(3) $f(x) = \sin(\cos x)$ (2π) (4) $f(x) = \sin^4 x + \cos^4 x$ $\left(\frac{\pi}{2}\right)$

(5) $f(x) = x - [x]$ $= \{x\}$ (One)

(6) Period of the function, $f(x) = [x] + [2x] + [3x] + ... + [nx] - \frac{n(n+1)}{2}x$ where $n \in N$ and [ ] denotes the greatest integer function, is

(A*) 1 (B) n (C) $\frac{1}{n}$ (D) non periodic

(7) $f(x) = \sin x + \cos ax$ is a periodic function then prove that 'a' must be rational

(8)(a) If $f(x) = \frac{\sin nx}{\sin\frac{x}{n}}$ has its period $= 4\pi$ then find the integral values of n.

(b) For $a > 0$, if $f(x + a) = \frac{1}{2} + \sqrt{f(x) - f^2(x)}$, prove that f is periodic.

(9) **prove:** $f(x) = \cos\sqrt{x}$; $x \sin x$ and $\sin x + \{x\}$ are aperiodic.

(10) $f(x) = 2\cos\left(\frac{x-\pi}{5}\right)$ (Ans: $p = 10\pi$)

(11) If $f(x) = (a + 3)x + 5a, x \in R$ is periodic. [Ans. $a = -3$]

(12) Consider those functions f that satisfy $f(x + 4) + f(x - 4) = f(x)$ for all real x. Any such function is periodic, and there is a least common positive period p for all of them. The value of p, is

(A) 8 (B) 12 (C) 16 (D*) 24

Composite of Function

(1)(a) $f(x) = \begin{cases} 1+x & \text{if } 0 \le x \le 2 \\ 3-x & \text{if } 2 < x \le 3 \end{cases}$ find fof

(b) $f(x) = \begin{cases} 1-x & \text{if } 0 \le x \le 1 \\ x+2 & \text{if } 1 < x < 2 \\ 4-x & \text{if } 2 \le x \le 4 \end{cases}$ find (fof) (x) [Ans: (fof) (x) = $\begin{cases} x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } 1 < x < 2 \\ 6-x & \text{if } 2 < x < 3 \\ x-3 & \text{if } 3 \le x \le 4 \\ 2 & \text{if } x = 2 \end{cases}$]

(c) $f(x) = \begin{cases} 1-x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$ and $g(x) = \begin{cases} -x & \text{if } x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$ find (fog)(x) and (gof)(x)

[Ans. (gof)(x) = $\begin{cases} x & \text{if } x \le 0 \\ -x^2 & \text{if } 0 < x < 1 \\ 1-x^2 & \text{if } x \ge 1 \end{cases}$; (fog)(x) = $\begin{cases} x^2 & \text{if } x < 0 \\ 1+x & \text{if } 0 \le x < 1 \\ x & \text{if } x \ge 1 \end{cases}$]

(d) $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & 0 \le x \le 1 \\ 2-x & x > 1 \end{cases}$ find (fof) (x)

(e) $\left.\begin{aligned} f(x) &= -1 + |x-2|, & 0 \le x \le 4 \\ g(x) &= 2 - |x|, & -1 \le x \le 3 \end{aligned}\right\}$ find gof and fog

[Ans: gof(x) = $\begin{cases} 1+x & 0 \le x \le 1 \\ 3-x & 1 < x < 2 \\ x-1 & 2 \le x < 3 \\ 5-x & 3 \le x \le 4 \end{cases}$; (fog) (x) = $\begin{cases} -1 & x = 0 \\ -(1+x) & -1 \le x < 0 \\ x-1 & 0 \le x \le 2 \end{cases}$]

(f) $f(x) = \begin{cases} 1+x^3 & x < 0 \\ x^2 - 1 & x \ge 0 \end{cases}$ and $g(x) = \begin{cases} (x-1)^{1/3} & x < 0 \\ (x+1)^{1/2} & x \ge 0 \end{cases}$ find g (f(x))

(g)(i) If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$ $(x \ne 0)$ find f (2). [Ans. $-\frac{7}{4}$]

(ii) Let f be a real valued function of real and positive argument such that

$$f(x) + 3x\, f\left(\frac{1}{x}\right) = 2(x+1) \text{ for all real } x > 0. \text{ The value of } f(10099) \text{ is}$$

(A) 550 (B) 505 (C*) 5050 (D) 10010

(h) Let $f : R \to R$ be a function such that $\left(f(x^3+1)\right)^{\sqrt{x}} = 5$, $\forall\ x \in (0, \infty)$ then the value of

$\left(f\left(\frac{27+y^3}{y^3}\right)\right)^{\sqrt{\frac{27}{y}}}$ for $y \in (0, \infty)$ is equal to

(A) 5 (B) 5^2 (C*) 5^3 (D) 5^6