by:- sudhir jainam

Super-30
(NM-I)

MATHEMATICS
Daily Practice Problems
Target IIT JEE 2020

Class:XI | *Topic : Even & Odd Function* | **DPP. NO.-19**

Find whether following functions are even / odd / neither (1 to 20) :

1. $y = x^4 - 2x^2$ 2. $y = x - x^2$ 3. $y = \cos x$ 4. $y = 2^x$ 5. $f(x) = Sgn(x) + x^{2000}$ 6. $f(x) = x^3 \cot x$

7. $f(x) = \tan(x^5)\, e^{x^3 \, Sgn\, x^7}$ 8. $f(x) = \sqrt[3]{1-x^2} + \sqrt[3]{1+x^2}$ 9. $f(x) = \ln\left(\dfrac{1+x^3}{1-x^3}\right)$

10. $f(x) = \dfrac{a^x - 1}{x^n(a^x + 1)}, \quad n \in I$ 11. $f(x) = \sin \log_{10}(x + \sqrt{1+x^2})$ 12. $f(x) = \dfrac{(1+2^x)^2}{2^x}$

13. $f(x) = \left\{(Sgn(x))^{Sgn\, x}\right\}^n, \quad n \in I$ 14. $f(x) = \dfrac{x}{e^x - 1} + \dfrac{x}{2} + 1$ 15. $f(x) = (-1)^{[x]}$

16. $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+21\pi}{\pi}\right] - 41}$ 17. $f(x) = \sin\left(\tan^{-1}\left(\sin^{-1}\left(\ln\left(x + \sqrt{x^2 + 1}\right)\right)\right)\right)$

18. $f(x) = \begin{cases} x|x| & x \le -1 \\ [1+x] + [1-x] & -1 < x < 1 \\ -x|x| & x \ge 1 \end{cases}$ 19. $f(x) = \begin{cases} \dfrac{x^{2n}}{(x^{2n}\, Sgn\, x)^{2n+1}}\left(\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right) & x \ne 0 \\ 1 & x = 0 \end{cases}$

20. $f(x) = \begin{cases} x^2 \sin \dfrac{\pi\, x}{2} & |x| < 1 \\ x|x| & |x| \ge 1 \end{cases}$

write following functions as sum of even and odd functions (21 to 23) :

21. $y = 1 - x^3 - x^4 - 2x^5$ 22. $y = \sin 2x + \cos \dfrac{x}{2} + \tan x.$ 23. $y = a^x$

24. If 'f' is an even function defined on the interval (-5, 5), then 4 real values of x satisfying the equation
$f(x) = f\left(\dfrac{x+1}{x+2)}\right)$ are ....,..............,.......... and ....

25. $f(x) = \int_0^x \log_e\left(\dfrac{1-x}{1+x}\right) dx$, then f(x) is ....(even / odd/ neither.)

26. If a function f(x) is even and odd simultaneously. Find f(3)–f(2).

27. Prove that the function defined as, $f(x) \begin{bmatrix} e^{-\sqrt{|\ln\{x\}|}} - \{x\}^{\sqrt{\frac{1}{|\ln\{x\}|}}} & \text{where ever it exists} \\ \{x\} & \text{otherwise, then} \end{bmatrix}$

f(x) is odd as well as even. (where{x}denotes the fractional part function)

28. Let $f:[-2,2] \to R$ where $f(x) = x^3 + \sin x + \left[\dfrac{x^2+1}{a}\right]$ be an odd function. Find the value of the parameter 'a'.
[Here[.] denotes greatest integer function].

Find even and odd extensions (29 to 31) :

29. $f(x) = x^2 + 5x - 2$ $x \in [0, 2]$ 30. $f(x) = x^3 \cot x + a^x$ $x \in [-4, 0]$

31. $f(x) = 3x^9 + x^2 + \cos x + \log(1 + |x|)$ $x \in [0, 5]$

32. Let $f(x) = \begin{cases} [x] \ ; x \geq 0 \\ g(x) ; x < 0 \end{cases}$ then find $g(x)$ if $f(x)$ is even

33. Let $f : [-2, 2] \to R$ be a function defined on $[0, 2]$ as follows.

$$f(x) = \begin{cases} x \tan x & ; \ 0 < x \leq \dfrac{\pi}{2} \\ \dfrac{\pi}{2}[x] & ; \ \dfrac{\pi}{2} < x \leq 2 \end{cases}$$ Define $f(x)$ on $[-2, 0)$ so that it becomes an

     (i) An even function on $[-2, 2]$,   (ii) An odd function on $[-2, 2]$.

34. If $f(x)$ is a function such that $f(x + y) = f(x) + f(y) \ \forall \ x, y \in R$ then prove that $f(x)$ is odd.

35. If $f(x)$ is a function such that $f(x + y) + f(x - y) = 2f(x) \cdot f(y)$ then prove that $f(x)$ is even.

36. Let $g(x + y) = g(x) \cdot g(y) \ \forall \ x, \ y \in R$, and $f(0) \neq 0$, $f(x) = \dfrac{g(x)}{1 + \{g(x)\}^2}$ then prove that $f(x)$ is even.

37. If $f(x)$ is a function such that $f(x) + f(y) = f\left(\dfrac{x + y}{1 - xy}\right) \ \forall \ x, \ y \in R, \ xy \neq 1.$, prove that $f(x)$ is odd

---

Answer Key

---

1. even **2.** neither even nor odd **3.** even **4.** neither even nor odd **5.** neither even nor odd

6. Even   **7.** Odd   **8.** Even   **9.** Odd   **10.** even if n is odd and odd if n is even

11. odd   **12.** Even   **13.** odd if n is odd and even if n is even   **14.** even

15. neither even nor odd   **16.** odd   **17.** odd   **18.** even   **19.** even   **20.** odd

21. $y = (1 - x^4) + (-x^3 - 2x^5)$ **22.** $y = (\sin 2x + \tan x) + \cos \dfrac{x}{2}$ **23.** $y = \dfrac{a^x + a^{-x}}{2} + \dfrac{a^x - a^{-x}}{2}$

24. $\dfrac{-3 - \sqrt{5}}{2}, \dfrac{-3 + \sqrt{5}}{2}, \dfrac{-1 - \sqrt{5}}{2}, \dfrac{-1 + \sqrt{5}}{2}$ **25.** Even   **26.** 0   **28.** $a > 5$ **29.** (i) $x^2 - 5x - 2$,

(ii) $-x^2 + 5x + 2$ **30.** (i) $x^3 \cot x + a^{-x}$, (ii) $-x^3 \cot x + a^x$

31. (i) $-3x^9 + x^2 + \cos x + \log(1 + |x|)$, (ii) $3x^9 - x^2 - \cos x - \log(1 + |x|)$.   **32.** $[-x]$

33. (i) $f(x) = \begin{cases} \dfrac{\pi}{2}[-x] & ; -2 \leq x < -\dfrac{\pi}{2} \\ x \tan x & ; -\dfrac{\pi}{2} \leq x < 0 \end{cases}$ (ii) $f(x) = \begin{cases} -\dfrac{\pi}{2}[-x] & ; -2 \leq x < -\dfrac{\pi}{2} \\ -x \tan x & ; -\dfrac{\pi}{2} \leq x < 0 \end{cases}$ PAGE#2