Find Range of following function :-

1. $y = x^2 - x + 2, \quad x \in [-2, 3]$
2. $y = 2x^2 + 5x - 1, \quad x \in [0, 3]$
3. $y = x^2 - 3x + 1$
4. $y = \sin^2 x - 2\sin x + 2$
5. $f(x) = \cos^2 x - 5\cos x - 6$
6. $f(x) = 4^x + 2^x + 1$
7. $y = x^4 - 3x^2 + 2$
8. $y = 2\{x\}^2 - \{x\} - 2$
9. $f(x) = \log_3(5 + 4x - x^2)$
10. $f(x) = \sin^{-1}(\sqrt{x^2 + x + 1})$
11. $f(x) = \log_3\left(\log_{0.5}\left(x^2 + 4x + 4\right)\right)$
12. $f(x) = \sqrt{x-1} + \sqrt{5-x}$
13. $f(x) = \sqrt{6-x} + 2\sqrt{x-4}$
14. $f(x) = \log_{0.5}\left(\sqrt{x-1} + \sqrt{5-x}\right)$
15. $f(x) = \left[\ln\left(\sin^{-1}\sqrt{x^2 + x + 1}\right)\right]$, where [.] denotes the greatest integer function.
16. $y = \log\sqrt{x^2 + 6x + 10}$
17. $f(x) = \dfrac{1}{2 - \sin 3x}$
18. $f(x) = \dfrac{1}{2 - \cos 5x}$
    (a) (1/3, 1) (b) [1/3, 1] (c) [1/3, 1] (d) none of these
19. $f(x) = \sqrt{9 - x^2}$
20. $f(x) = \dfrac{x^2 - 2}{x^2 - 3}$
21. $f(x) = \dfrac{1 + x^2}{x^2}$
    (a) (0, 1) (b) [0, 1] (c) $(1, \infty)$ (d) $[1, \infty)$
22. $y = \dfrac{x}{4 - x^2}$
23. $y = \sin^{-1}\left[\dfrac{1}{2} + x^2\right]$
24. $y = \dfrac{1}{\sqrt{4 + 3\cos x}}$
25. $y = \dfrac{1}{\sqrt{x - [x]}}$
26. $f(x) = \left[\dfrac{1}{\sin\{x\}}\right]$,
    (a) Z (b) N
    (c) $\{x : x \geq 0, x \in Z\}$ (d) $\{x : x \geq 2, x \in N\}$
27. $f(x) = \log\left[\cos|x| + \dfrac{1}{2}\right]$
28. $f(x) = \dfrac{x - [x]}{1 - [x] + x}$
29. $f(x) = \sin^{-1}\left(\dfrac{x^2 + 1}{x^2 + 2}\right)$
30. $f(x) = \sqrt{\log_3(\cos(\sin x))}$
31. $f(x) = \dfrac{x^2 + 2x + 3}{x}$
32. $f(x) = \dfrac{x - 1}{x^2 - 2x + 3}$
33. $y = \dfrac{x^2 + x + 2}{x^2 + x + 1}$
34. $y = \dfrac{x + 2}{x^2 - 8x - 4}$
35. $y = \dfrac{x - 2}{x^2 - 2x + 3}$
36. $y = \dfrac{x}{1 + x^2}$
37. $y = \dfrac{x^2 - x}{x^2 + 2x}$
38. $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$
    (a) R–[1/5,1] (b) R (c) R – {1} (d) none of these
39. $e^x + e^{f(x)} = e$,
    (a) $(-\infty, 1)$ (b) $(-\infty, 0)$ (c) $(1, \infty)$ (d) none of these
40. $f(x) = |x-1| + |x-2|, -1 \leq x \leq 3$
41. $y = |x-2| + 2|2x+1| \quad 0 \leq x \leq 5$
42. $y = |x-1| + |x-2| + |x-3|$
43. $y = 3|x-3| - 2|x+2|$
44. $f(x) = \sin x + \cos x + 3$
45. $y = 3\sin x - 4\cos x + 2$

46. $y = 3\sin x + 4\cos\left(x + \frac{\pi}{3}\right) + 7$

47. $f(x) = |\sin x| - \cos x$

48. $f(x) = 2|\cos x| + 3|\sin x|$

49. $f(x) = [\sin x]$

50. $f(x) = [\sin x + \cos x]$

51. $f(x) = [\,|\sin x| + \cos x\,]$

52. $f(x) = [\,\sin x - |\cos x|\,]$

53. $f(x) = [|\sin x| + |\cos x|]$, where [.] denotes the greatest integer function.

54. $f(x) = \sqrt{[\sin 2x] - [\cos 2x]}$, where [.] denotes greatest integer.

55. $f(x) = [\,|\sin x - \cos x|\,]$

56. Find domain and range of $f(x) = \log_e\left(\frac{1}{\sqrt{[\cos x] - [\sin x]}}\right)$, where [.] denotes the greatest integer function.

57. $f(x) = {}^{7-x}P_{x-3}$

(a) {1, 2, 3} (b) {1, 2, 3, 4, 5, 6}
(c) {1, 2, 3, 4} (d) {1, 2, 3, 4, 5}

58. $f(x) = {}^{9-x}C_{2x-1}$

59. $f(x) = {}^{15-2x}P_{x+1} + {}^{8-x}C_{3x-1}$

60. $f(x) = \sqrt{a^2\cos^2 x + b^2\sin^2 x} + \sqrt{a^2\sin^2 x + b^2\cos^2 x}$, where $a \neq b$.

61. $f(x) = \frac{\sin^2 x + \sin x - 1}{\sin^2 x - \sin x + 2}$

62. $f(x) = \ln(2\sin x + \tan x - 3x + 1)$, where $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

63. $f(x) = \cos^{-1}\left(\frac{x^2}{\sqrt{1+x^2}}\right)$

64. $f(x) = \left(\tan^{-1} x\right)^2 + \frac{2}{\sqrt{1+x^2}}$

65. $f(x) = \cos\left(\sin\left(\ln\left(\frac{x^2+e}{x^2+1}\right)\right)\right) + \sin\left(\cos\left(\ln\left(\frac{x^2+e}{x^2+1}\right)\right)\right)$

66. $f(x) = \frac{\tan(\pi[x^2 - x])}{1 + \sin(\cos x)}$, where [.] denotes the greatest integer function.

67. Given that y = f(x) is a function whose domain is [4, 7] and range is [–1, 9]. Find the range and somain of

(a) $g(x) = \frac{1}{3}f(x)$ (b) $h(x) = f(x - 7)$

68. $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$

69. $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} - \frac{\cos x}{\sqrt{1+\cot^2 x}}$

70. If $g(x) = \left(\frac{1}{1-x}\right)$ and $g_2(x)$ denotes g(g(x)), and $g_3(x)$ denotes g(g(g(x))). Then find the range of $g_{3n+1}(x)$. $n \in I^+$

71. $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] denotes the greatest integer function.

72. $f(x) = [x^2] - [x]^2$, $x \in [0, 2]$, where [.] denotes greatest integer function.

73. $f(x) = \sec^{-1}(\log_3 \tan x + \log_{\tan x} 3)$

74. Find the range of the function

$f(x) = \sqrt{-x^2 + 4x - 3} + \sqrt{\sin\left(\frac{\pi}{2}\left(\sin\frac{\pi}{2}(x-1)\right)\right)}$

75. Find domain and range of

$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{\ldots\ldots\ldots\infty}}}}$

76. $f(x) = [1 + \sin x] + [1 + \sin 2x] + \ldots\ldots\ldots + [1 + \sin nx]$, $n \in N$. (where [.] denotes greatest integer function)

77. $f(x) = [[x] - 1] + \cos^2 x$, where [.] denotes the greatest integer function.

78. $f(x) = \log_{[x-1]} \sin x$, where [.] denotes greatest integer.

79. $f(x) = \frac{e^x}{[x+1]}, x \geq 0$, where [.] denotes the greatest integer function.

80. Let $f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 1 \\ 3 - x, & 1 < x < \infty \end{cases}$

Define $f(f(x))$. Also obtain domain and range of $f(f(x))$.

81. Functions f(x) and g(x) are defined in [a, b] such that f(x) is monotonically increasing while g(x) is monotonically decreasing. It is given that the range of f(x) and g(x) are subsets of [a, b]. Find domain and range of h(x) = fog(x) + gof(x).

82. Find the domain and range of h(x) = g(f(x)), where

$f(x) = \begin{cases} [x], & -2 \leq x \leq -1 \\ |x| + 1, & -1 < x \leq 2 \end{cases}$ and

$f(x) = \begin{cases} [x], & -\pi \leq x \leq 0 \\ \sin x, & 0 < x \leq \pi \end{cases}$

where [.] denotes greatest integer function.

83. Find all possible values of real parameter 'a' so that the range of $f(x) = \frac{x-1}{a - x^2 + 1}$ does not contain any value belonging to the interval $\left[-1, -\frac{1}{3}\right]$.

84. Let $f(x)=\begin{cases} x^2-4x+3 & , x<3 \\ x-4 & , x\geq 3 \end{cases}$ and

$g(x)=\begin{cases} x-3 & , x<4 \\ x^2+2x+2 & , x\geq 4 \end{cases}$

Describe the function f/g and find its domain and range.

85. Let $f(x)=\sqrt{ax^2+bx}$. Find the set of real values of 'a' for which there is at least one positive rea value of 'b' for which the domain of *f* and the range of *f* are the same set.

ANSWER KEY

5. $R_f=[-10, 0]$

6. $R_f=(1, \infty)$

9. $R_f=(-\infty, 2]$

10. $R_f=[\pi/3, \pi/2]$

11. R

12. $[2, 2\sqrt{2}]$

13. $R_f=\left[\sqrt{2}, \sqrt{10}\right]$

14. $R_f=[-3/2, -1]$

15. $R_f=\{0\}$

17. C

18. $R_f=[1/3, 1]$

19. $R_f=[0,3]$

20. $R_f=(-\infty, 2/3]\cup(1, \infty)$

21. C

26. B

27. {0}

28. $\left[0, \frac{1}{2}\right)$

29. $R_f=[\pi/6, \pi/2)$

30. $R_f=\{0\}$

31. $\left(-\infty, 2-2\sqrt{2}\right]\cup\left[2+2\sqrt{3}, \infty\right)$

32. $\left[\frac{1}{4}(1-\sqrt{3}), \frac{1}{4}(1+\sqrt{3})\right]$

38. C

39. A

40. Range $\in[1,5]$

44. Range $\in[3-\sqrt{2}, 3+\sqrt{2}]$

53. {1}

54. [0, 1]

56. $D_f=\left[2n\pi-\frac{\pi}{2}, 2n\pi\right], R_f=\{0\}$

57. A

60. $R_f=\left[a+b, \sqrt{2\left(a^2+b^2\right)}\right]$

61. $R_f=\left[\left(2-2\sqrt{11}\right)/7, 1/2\right]\cup\{1\}$

62. $R_f=\left[\ln\left(2+\frac{1}{\sqrt{3}}-\frac{\pi}{2}\right), \ln\left(2\sqrt{3}+1-\pi\right)\right]$

63. $R_f=(0, \pi/2]$

64. $R_f=[2, \pi^2/4)$

65. $R_f=\left[\cos(\sin 1)+\sin(\cos 1), 1+\sin 1\right]$

66. $R_f=\{0\}$

67. (a) Range : [–1/3, 3], Domain = [4, 7]
(b) Range [–1, 9] and domain [11, 14]

68. $R_f=[0, 1/2]$

69. [-1 , 1]

71. $R_f=\{\pi\}$

72. $R_f=\{0,1,2\}$

73. $R_f=[\pi/3, \pi/2)\cup(\pi/2, 2\pi/3]$

74. Range of [0, 2]

75. $D_f=R, R_f=[0, 2]$

76. $R_f=\{0, 1, 2, 3, \ldots, n+1\}$

77. $R_f=[-1, 0]\cup\left\{\cos^2\lambda\right\}, \lambda\in I$

78. $R_f=(-\infty, 0]$

79. $R_f=[1, \infty)$

80. $D_f=[0,3], R_f=[1,2]$

81. $D_h=[a,b], R_h=[h(a), h(b)]$

82. $D_h=[-2,2], R_h=\{-2,-1\}\cup[\sin 3, 1]$

83. $a\in(-\infty, -1/4)$

84. $(f/g)x=\begin{cases} x-1 & , x<3 \\ \dfrac{x-4}{x-3} & , 3<x<4 \\ \dfrac{x-4}{x^2+2x+2} & , x\geq 4 \end{cases}$ $D(f/g)=R-\{3\}$

85. $a\in\{0, -4\}$. [-1, 1]