

Super-30

(NM-I)

The domain of definition of the function :

1. $y = \sqrt{2x - x^2}$

2. $f(x) = \sqrt{x^2 - 5x + 6}$

3. $y = \sqrt{\frac{x+3}{5-x}}$

4. $y = \sqrt{-4x^2 + 4x + 3}$

5. $y = \sqrt{6+7x-3x^2}$

6. $y = \sqrt{\frac{1}{2x^2 - 5x - 3}}$

7. $f(x) = \sqrt{4x - x^3}$

8. $f(x) = \sqrt{3x - x^3}$

9. $f(x) = \sqrt{\frac{(2x+1)}{x^3 - 3x^2 + 2x}}$

10. $y = \sqrt{5-x - \frac{6}{x}}$

11. $y = \sqrt{\frac{x^2 - 7x + 12}{x^2 - 2x - 3}}$

12. $y = \sqrt{\frac{x^2 - 5x + 6}{x^2 + 6x + 8}}$

13. $y = \log_2 \frac{x-2}{x+2}$

14. $f(x) = \log \frac{x+3}{x+1}$

15. $y = \log \frac{x^2 + 8x + 7}{x^2 + 7}$

16. $y = \log((x^2 - 3x)(x+5))$

17. $f(x) = \ln \left(\frac{20}{(x-3)(x-4)} + \frac{10}{x-4} + 1 \right)$

18. $f(x) = \sqrt{\left(\frac{2}{x^2 - x + 1} - \frac{1}{x+1} - \frac{2x-1}{x^3 + 1} \right)}$

19. $f(x) = \sqrt{(x-1)(x-2)(3-x)(x+1)(2-x)}$

20. $f(x) = \sqrt{\left(\frac{1}{x^2 + 1} + \frac{2}{x^3 - x^2 + x - 1} + \frac{1}{x^3 + x^2 + x + 1} \right)}$

21. $f(x) = \sqrt{\frac{(x-4)(x-5)(x-2)}{x^4 - 2x^3 - x^2 + 2x}}$

22. $y = \sqrt{x-1}\sqrt{x+1}$

23. $y = \sqrt{x-1} + \sqrt{6-x}$

24. $f(x) = \sqrt{2-x} + \sqrt{1+x}$

25. $f(x) = \sqrt{x^2 - x - 20} + \sqrt{6-x}$

26. $y = \sqrt{1-x^2} + \sqrt{3x-x^2-2}$

27. $y = \sqrt{x^2 - x - 20} + \frac{1}{\sqrt{x^2 - 5x - 14}}$

28. $y = \frac{1}{\sqrt{14+5x-x^2}} + \sqrt{x^2 - x - 20}$

29. $y = \frac{1}{x-1} + \sqrt{2+x}$

30. $y = \frac{\sqrt{4-3x-x^2}}{x+4}$

31. $f(x) = \frac{\sqrt{12+x-x^2}}{x(x-2)}$

32. $f(x) = \frac{\sqrt{x^2 + x - 6}}{x^2 - 4}$

34. $f(x) = \log_x e$

36. $\sqrt{4-x} + \frac{1}{\log_{10} x}$

38. $y = \frac{\sqrt{x+5}}{\log(9-x)}$

40. $y = \sqrt{x^2 + 4x - 5} \cdot \log(x+1)$

42. $y = \sqrt{x^2 + 4x - 5} \cdot \log(x+5)$

44. $y = \log(5x^2 - 8x - 4) + (x+3)^{-0.5}$

46. $y = \sqrt{16x - x^5} + \log_{1/2}(x^2 - 1)$

48. $y = \sqrt{\log(x+1)}$

50. $y = \sqrt{\log \frac{1-2x}{x+3}}$

52. $f(x) = \sqrt{\log_{0.4}(x - x^2)}$

54. $y = \sqrt{\log_{0.5}(x^2 - 9) + 4}$

56. $\sqrt{1 - \log_3 x}$

58. $f(x) = \sqrt{\log_{1/2} \frac{x}{x^2 - 1}}$

60. $f(x) = \cos^{-1}\left(\frac{6 - 3x}{4}\right) + \operatorname{cosec}^{-1}\left(\frac{x-1}{2}\right)$

61. $\frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$

63. $y = \sqrt{3-x} + \arcsin \frac{3-2x}{5}$

65. $f(x) = \arcsin 3^x$

33. $y = \frac{\sqrt{x+12-x^2}}{x^2 - 9}$

35. $\frac{1}{\log_{10}(1-x)} + \sqrt{(x+2)}$

37. $y = \frac{1}{3 - \log_3(x-3)}$

39. $y = \sqrt{4x - x^2} - \log_3(x-2)$

41. $f(x) = \log(5x^2 - 8x - 4) + \sqrt{x-1}$

43. $y = \frac{\log(3 - 2x - x^2)}{\sqrt{x}}$

45. $y = \sqrt{4x - x^2} + \log(x^2 - 1)$

47. $f(x) = \frac{1}{\sqrt{4x^2 - 1}} + \ln x(x^2 - 1)$

49. $y = \sqrt{\log \frac{3-x}{x}}$

51. $y = \sqrt{\log_{0.3} \frac{x-1}{x+5}}$

53. $y = \sqrt{\log_{0.3}(x^2 - 5x + 7)}$

55. $y = \sqrt{\log_{0.4} \frac{x-1}{x+5} \cdot \frac{1}{x^2 - 36}}$

57. $f(x) = \sqrt{\log[(5x - x^2)/6]}$

59. $f(x) = \frac{1}{\sqrt{\log_{0.5}(x^2 - 7x + 13)}}$

62. $y = \arcsin \frac{x-3}{2} - \log(4-x)$

64. $f(x) = \sin^{-1} \left(\log_2 \left(\frac{x^2}{2} \right) \right)$

66. $\log \cos x$

67. $f(x) = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$ 68. $y = \sqrt{3 \sin x - 1}$

69. $y = \sqrt{2 \sin \frac{x}{2}}$

70. $y = \frac{1}{\sqrt{4 \cos x + 1}}$ 71. $y = \sqrt{-2 \cos^2 x + 3 \cos x - 1}$

72. $y = \sqrt{\sin^2 x - \sin x}$ 73. $y = \arccos \frac{2}{2 + \sin x}$

74. $f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$

75. $f(x) = \sqrt{2 - [x]} + \sqrt{1 + [x]}$

76. Write explicitly, functions of y defined by the following equations and also find the domains of definition of the given implicit functions :

(a) $10^x + 10^y = 10$ (b) $x + |y| = 2y$

77. The function $f(x)$ is defined on the interval $[0, 1]$. Find the domain of definition of the functions

(a) $f(\sin x)$ (b) $f(2x+3)$

78. The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(\tan x)$, $f(3 \sin x)$, $f(\log x)$, $f(|x-2|)$

79. $f(x) = \log |4 - x^2|$ 80. $f(x) = \ln(\sqrt{x^2 - 5x - 24} - x - 2)$

81. $f(x) = \frac{1}{\sqrt{x - |x|}}$ 82. $f(x) = \frac{1}{\sqrt{|\cos x| + \cos x}}$

83. $f(x) = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$ 84. $f(x) = \sqrt{e^{(\sqrt{2-x} + \sqrt{1+x})}}$

85. $f(x) = \cot^{-1}(e^x + e^{-x})$ 86. $f(x) = \sqrt{\frac{1 - 5^x}{7^{-x} - 7}}$

87. $f(x) = \sqrt{\frac{3^x - 4^x}{2x^2 - x - 8}}$ 88. $f(x) = \sqrt{\frac{\log_{0.3}(x-1)}{x^2 - 2x - 8}}$

89. $f(x) = \sqrt{(x^2 - 3x - 10) \ln^2(x-3)}$

90. $f(x) = \sqrt{77^x - 777^x} + \sqrt{3-x} + \sin^{-1}\left(\frac{3-2x}{5}\right)$

91. $f(x) = \sqrt{\cos 2x} + \sqrt{16-x^2}$ 92. $y = \log_{10} \sin(x-3) + \sqrt{16-x^2}$

93. $f(x) = \sqrt{\log_x(\cos 2\pi x)}$, $f(x) = \log_x \sin x$

94. $y = \frac{\sqrt{\cos x - 1/2}}{\sqrt{6 + 35x - 6x^2}}$ 95. $f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)} + \sqrt{\cosec x} + (\sin x)^{\frac{1}{3}}$

96. $f(x) = \sqrt{\cos(\sin x)} + \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1+x}{1-x}}$

97. $f(x) = \sin|x| + \sin^{-1}(\tan x) + \sin(\sin^{-1}x)$ 98. $f(x) = \sqrt{x^2 - |x|} + \frac{1}{\sqrt{9-x^2}}$

99. $f(x) = \sqrt{\frac{x}{1-|x|}} + \log_e(\sqrt{x+2} + \sqrt{2-x})$

100. $f(x) = \frac{1}{x} + 3^{\cos^{-1}x} + \frac{1}{\sqrt{x-2}}$ 101. $f(x) = \frac{1}{(|x|-1)\{\cos^{-1}(2x+1)\}\tan 3x}$

102. $f(x) = \sqrt{\{\cos(\sin x)\}} + \sin^{-1}\{(1+x^2)/(2x)\}$ 103. $f(x) = \frac{\log_{2x} 3}{\cos^{-1}(2x-1)}$

104. $f(x) = \log_{10} \log_{10} \log_{10} \log_{10} \dots \log_{10} x$ 105. $f(x) = \log_{100x} \left(\frac{2 \log_{10} x + 1}{-x} \right)$

106. $f(x) = \underbrace{\log_{10} \log_{10} \log_{10} \log_{10} \dots \log_{10} x}_{n-times \log}$

107. $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

108. $f(x) = \log_{10}(1 - \log_{10}(x^2 - 5x + 16))$

109. $f(x) = \sqrt{\log_{0.3} \left(\frac{-3}{(x-1)(x-2)} + \frac{3}{x-2} + 2 \right)}$

110. $\psi(x) = \sqrt{\frac{(\log_{0.2} x)^3}{(\log_{0.2} x^3)(\log_{0.2} 0.0016x) + 36}} D_\psi.$

111. $f(x) = \log_2 \left(-\log_2 \left(1 + \frac{1}{\sqrt[4]{x}-1} \right) \right)$

112. $f(x) = 3^{\cos^{-1}(\log_e \sqrt{1-e^{2x}})}$

113. $f(x) = \log_{10} \log_2 \log_{2/\pi} (\tan^{-1} x)^{-1}$

114. $f(x) = \sqrt{\log_{1/3}(\log_4(|x|^2 - 5))}$

115. If a function is defined as $g(x) = |\sin x| + \sin x$, $\phi(x) = \sin x + \cos x$, $0 \leq x \leq \pi$
then find the domain for $f(x) = \sqrt{\log_{\phi(x)} g(x)}$.

116. $f(x) = \sqrt{x^2 + 4x} C_{2x^2+3}$

117. $f(x) = \sqrt{\log_{1/2} \left(\frac{5x-x^2}{4} \right) + 10} C_x$

118. (a) $f(x) = \sin^{-1}[2x^2 - 3]$, where $[.]$ denotes the greatest integer function.

(b) $f(x) = \log_{\left[\frac{x+1}{x}\right]} |x^2 - x - 6| + 16^{-x} C_{2x-1} + 20^{-3x} P_{2x-5}$

119. $f(x) = \operatorname{cosec}^{-1}[1 + \sin^2 x]$, where $[.]$ denotes greatest integer function.

120. $f(x) = \sin^{-1} \left(\frac{[x]}{\{x\}} \right)$, where $[.]$ and $\{.\}$ denotes the greatest integer and fractional part.

121. $f(x) = \cos^{-1} \sqrt{\log[x] \frac{|x|}{x}}$, where $[.]$ denotes the greatest integer.

122. $f(x) = \frac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10) + \frac{1}{\sqrt{2-|x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$

123. $f(x) = \frac{3}{[\frac{x}{2}]} - 5 \cos^{-1} x^2 + \frac{(2x+1)!}{\sqrt{x+1}}$, where $[.]$ denotes the greatest integer.

124. $f(x) = \frac{\sqrt{\sin^{-1} x + \sqrt{x^2 + 1}} + \sqrt{x - [x] + \log x}}{e^{\sqrt{\sin x + \cos x}} + \log \left(\sin \left(\frac{1}{\sqrt{-x^2}} \right) \right)}$

125. $f(x) = \sqrt{(5x-6-x^2)[\{\ln x\}]} + \sqrt{(7x-5-2x^2)} + \left(\ln \left(\frac{7}{2} - x \right) \right)^{-1}$

126. $f(x) = \log_{(\cosec x - 1)}(2 - [\sin x] - [\sin x]^2)$

127. If domain for $y = f(x)$ is $[-3, 2]$, find domain of $g(x) = f\{ |[x]| \}$.

128. $f(x) = \frac{x}{\sqrt{\sin(\ln x) - \cos(\ln x)}}$

129. $f(x) = (\pi^{3-x} - e^{2x-1})^{\frac{1}{2002}} + \sqrt{\cos(\sin x)}$

130. $f(x) = \log(e^x - x)$

131. $f(x) = \sqrt{\sqrt{\sin x} + \sqrt{\cos x}} - 1$

132. $f(x) = \frac{1}{[|x-2|] + [|x-10|] - 8}$

133. Find the domain of $\sqrt{[f(x)] + 1 \cdot 2}$, where $f(x) = 2 \left| \sqrt{x^2 - 2x + 1} - e^{\int_0^1 \left(\frac{1}{x+1} \right) dx} \right|$

134. Find domain and range of

$$f(x) = \sqrt{(1 - \cos x) \sqrt{(1 - \cos x) \sqrt{(1 - \cos x) \sqrt{\dots \dots \dots \infty}}}$$

135. Let $f(x) = \log_2 \left(\log_3 \left(\log_4 \left(\log_5 (\sin x + b^2) \right) \right) \right)$, where $b \in \mathbb{R}$. Find values of 'b' so that $f(x)$ has domain $(-\infty, \infty)$.

136. Let $f(x) = \operatorname{sgn} \left(\log_{0.5} \left(\log_6 \left(\frac{x^2 + x}{x + 4} \right) \right) \right)$ and range of $f(x) \in \{-1\}$. Find domain of $f(x)$.

$$137. \quad f(x) = \frac{[x]}{2x - [x]}$$

ANSWER

1. $D(y) = [0, 2]$ 2. $x \in (-\infty, 2] \cup [3, \infty)$ 3. $D(y) = [-3, 5)$ 4. $D(y) = [-1/2, 3/2]$
 5. $D(y) = [-2/3, 3]$ 6. $D(y) = (-\infty, -1/2) \cup (3, +\infty)$ 7. $D(f) = (-\infty, -2] \cup [0, 2]$
 8. $D(f) = (-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$ 9. $x \in (-\infty, -1/2] \cup (0, 1) \cup (2, \infty)$ 10. $D(y) = (-\infty, 0) \cup [2, 3]$
 11. $D(y) = (-\infty, -1) \cup [4, +\infty)$ 12. $D(y) = (-\infty, -4) \cup (-2, 2) \cup (2, +\infty)$ 13.
 $D(y) = (-\infty, -2) \cup (2, +\infty)$ 14. $D(y) = (-\infty, -3) \cup (-1, +\infty)$ 15. $D(y) = (-\infty, -7) \cup (-1, +\infty)$
 16. $D(y) = (-5, 0) \cup (3, +\infty)$ 17. $x \in (-\infty, -2) \cup (-1, 3) \cup (4, \infty)$ 18. $x \in (-\infty, -1) \cup (1, 2]$
 19. $[-1, 1] \cup [3, \infty) \cup \{2\}$ 20. $(-\infty, -3] \cup (-1, 0] \cup [1, \infty)$ 21. $(-1, 0) \cup (1, 4] \cup [5, \infty) - \{2\}$
 22. $D(y) = [-1, 1]$ 23. $D(y) = [1, 6]$ 24. $D(f) = [-1, 2]$ 25. $D(f) = (-\infty, -4] \cup [5, 6]$
 26. $D(y) = \{1\}$ 27. $D(y) = (-\infty, -4) \cup (7, +\infty)$ 28. $D(y) = [5, 7)$
 29. $D(y) = [-2, 1] \cup (1, +\infty)$ 30. $D(y) = (-4, 1]$ 31. $(-3, 0) \cup (0, 2)$
 32. $D(f) = (-\infty, -3] \cup (2, +\infty)$ 33. $D(y) = (-3, 3) \cup (3, 4]$ 34. $[0, \infty[- \{-1\}$
 35. $(-2 \leq x < 0) \cup (0 < x < 1)$ 36. $x \in (0, 4] - \{1\}$ 37. $D(y) = (3, 30) \cup (30, +\infty)$
 38. $D(y) = (-5, 8] \cup (8, 9)$ 39. $D(y) = (2, 4]$ 40. $D(y) = [1, +\infty]$
 41. $D(f) = [2, +\infty]$ 42. $D(y) = [1, +\infty)$ 43. $D(y) = (0, 1)$
 44. $D(y) = (-3, -2/5) \cup (2, +\infty)$ 45. $D(y) = (1, 4]$ 46. $(-\infty, -2] \cup]1, 2]$
 47. $(-1 < x < -1/2) \cup (x > 1)$ 48. $D(y) = [0, +\infty)$ 49. $D(y) = (0, 3/2]$
 50. $D(y) = (-3, -2/3]$ 51. $D(y) = (1, +\infty)$ 52. $(0, 1)$
 53. $D(y) = [2, 3]$ 54. $D(y) = [-5, -3] \cup (3, 5)$ 55. $D(y) = (1, 6) \cup (6, +\infty)$ 56. $x \in (0, 3]$
 58. $\left[\frac{1-\sqrt{5}}{2}, 0 \right) \cup \left[\frac{1+\sqrt{5}}{2}, \infty \right)$ 59. $x \in (3, 4)$ 60. $\left[3, \frac{10}{3} \right]$
 61. $[1, 2)$ 62. $D(y) = [1, 4)$ 63. $D(y) = [-1, 3]$
 64. $[-2, -1] \cup [1, 2]$ 65. $D(y) = (-\infty, 0]$ 66. $\{x : 2n\pi - \pi/2 < x < 2n\pi + \pi/2\}, n \in \mathbb{Z}$
 67. $(2K\pi, (2k+1)\pi)$ 68. $D(y) = \{[2\pi n + \arcsin(1/3); \pi(2n+1) - \arcsin(1/3)] \mid n \in \mathbb{Z}\}$
 69. $D(y) = \{[4\pi n, 2\pi(2n+1)] \mid n \in \mathbb{Z}\}$
 70. $D(y) = \{(\pi(2n-1) + \arccos(1/4), \pi(2n+1) - \arccos(1/4)) \mid n \in \mathbb{Z}\}$
 71. $D(y) = \{[\pi(6n-1)/3, \pi(6n+1)/3] \mid n \in \mathbb{Z}\}$
 72. $D(y) = \{\pi(4n+1)/2, \mid n \in \mathbb{Z}\} \cup \{[\pi(2m+1), 2\pi(m+1)] \mid m \in \mathbb{Z}\}$
 73. $D(y) = \{[2\pi n, \pi(2n+1)] \mid n \in \mathbb{Z}\}$ 74. $\{2, 3\}$ 75. $[-1, 3)$

76.(a) $y = \log(10 - 10^x)$, $-\infty < x < 1$, (b) $y = x/3$ when $-\infty < x < 0$ & $y = x$ when $0 \leq x < +\infty$

77.(a) $2K\pi \leq x \leq 2K\pi + \pi$ where $K \in \mathbb{I}$ (b) $[-3/2, -1]$

$$78. f(\tan x) \in \left[n\pi, n\pi + \frac{\pi}{4} \right].$$

$$79. 4 - x^2 \neq 0 \quad 80. (-\infty, -3] \quad 81. \phi \quad 82. \left(\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2} \right) \quad 83. x \in [4, 6] \quad 84. [-1, 2]$$

$$85.R \quad 86. (-\infty, -1) \cup [0, \infty) \quad 87. \left(-\infty, \frac{1-\sqrt{65}}{4} \right) \cup \left[0, \frac{1+\sqrt{65}}{4} \right) \quad 88. x \in (2, 4) \quad 89. [5, \infty)$$

$$90. [-1, 0] \quad 91. \left[-\frac{5\pi}{4}, -\frac{3\pi}{4} \right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4} \right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4} \right] \quad 92. (3-2\pi < x < 3-\pi) \cup (3 < x \leq 4)$$

93. $(0, 1/4) \cup (3/4, 1) \cup \{x : x \in \mathbb{N}, x \geq 2\}$, $2K\pi < x < (2K+1)\pi$ but $x \neq 1$ where K is non-negative integer

$$94. [0, \pi/3] \cup [5\pi/3, 6] \quad 95. [1, \pi) \quad 96. \phi \quad 97. [-\pi/4, \pi/4] \quad 98. (-3, -1] \cup \{0\} \cup [1, 3)$$

$$99. [-2, -1) \cup [0, 1) \quad 100. \phi \quad 101. \left(-1, \frac{\pi}{6} \right) \cup \left(\frac{-\pi}{6}, 0 \right) \quad 102. x = -1 \text{ and } x = 1$$

$$103. x \in (0, 1/2) \cup (1/2, 1) \quad 104. (0, \infty) \quad 105. \left(0, \frac{1}{100} \right) \cup \left(\frac{1}{100}, \frac{1}{\sqrt{10}} \right) \quad 106. (10^{10^{10}}, \infty)$$

$$107. \left(-4, -\frac{1}{2} \right) \cup (2, \infty) \quad 108. x \in (2, 3) \quad 109. [-2, 1) \quad 110. [0, 25]$$

$$111. (0, 1) \quad 112. \left(-\infty, \frac{1}{2} \log_e \left(1 - \frac{1}{e^2} \right) \right] \quad 113. \phi \quad 114. [-3, -2) \cup [3, 4)$$

$$115. \text{Domain of } f(x) \text{ is } \left[\frac{\pi}{6}, \frac{\pi}{2} \right) \quad 116. x \in \{1, 2, 3\} \quad 117. x \in \{1, 4\}$$

$$118. \text{(a)} \left[-\sqrt{\frac{5}{2}}, -1 \right] \cup \left[1 - \sqrt{\frac{5}{2}}, \infty \right], \text{(b)} x \in \{4, 5\} \quad 119.R \quad 120. (0, 1) \quad 121. x \in (2, \infty)$$

$$122. (-2, -1) \cup (-1, 0) \cup (1, 2) \quad 123. x \in \{-1/2\} \quad 125. (1, 2) \cup (2, 5/2)$$

$$127. g(x) \in [-2, 3] \text{ or } [-2, 3). \quad 128. x \in \left(e^{\left(2n + \frac{1}{4} \right)\pi}, e^{\left(2n + \frac{5}{4} \right)\pi} \right) \quad 129. \left(-\infty, \frac{1 + 3 \ln \pi}{2 + \ln \pi} \right)$$

$$130.R \quad 131. \left[2K\pi, 2K\pi + \frac{\pi}{2} \right]$$

$$132. R - \{(1, 2) \cup \{2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup (10, 11)\} \quad 134. D_f = R, R_f = [0, 2]$$

$$135. b \in (-\infty, -\sqrt{626}) \cup (\sqrt{626}, \infty) \quad 136. D_f = (-4, -3) \cup (8, \infty) \quad 137. R - \left\{ -\frac{1}{2}, 0 \right\}$$