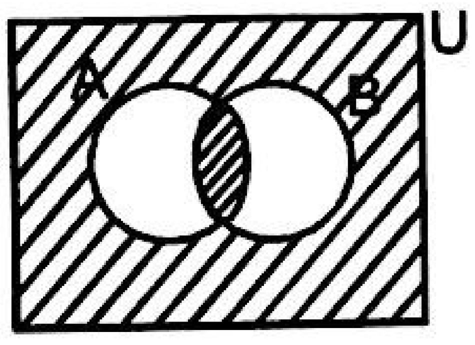
Board Level Exercise

Type (I) : Very Short Answer Type Questions : **[01 Mark Each]**

1. If U = Number of students in class XI of school S and A = Number of girls in class XI of school S. Find A′.

2. Represent the shaded part of the Venn diagram in terms of union and intersection of the sets A and B and their complements.



3. If the ordered pair (x – 1, – 5) = (2, y + 3) find x and y.

4. If A × B = {(a, 1), (a, 2), (a, 5), (b, 2), (b, 5), (b, 1)} then find B × A.

5. If A × B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)} then find A and B.

6. Let n(A) = n. Then find the number of all relations on A.

7. If R is a relation from a finite set A having m elements to a finite set B having n elements, then find the number of relations from A to B.

8. Show that the relation R in R defined as R = {(a, b) : a ≤ b} is not symmetric.

9. Show that the relation R in the set {1, 2, 3}, given by R = {(1, 2), (2, 1)} is not reflexive.

Type (II) : Short Answer Type Questions : **[02 Marks Each]**

10. If A = {1, 2, 3, 4, 5}, B = {1, 3, 5, 8}, C = {2, 5, 7, 8} verify that A – (B ∪ C) = (A – B) ∩ (A – C).

11. Given a universal set U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.
Find the complement of each of the following :
(i) A = {1, 3, 7, 9} (ii) B = {0} (iii) C = ϕ (iv) D = {3, 7, 8, 9}

12. If U = {1, 2, 3, 4, ... , 10} is the universal set for the sets A = {2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6} then verify that (A ∪ B)′ = A′ ∩ B′.

13. If U = {a, e, i, o, u}, A = {a, e, i}, B = {e, o, u} and C = {a, i, u} then verify that A ∩ (B – C) = (A ∩ B) – (A ∩ C)

14. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that (A ∩ B)′ = A′ ∪ B′.

15. In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee ?

16. List all the subsets of {–1, 0, 1}.

17. If A = {1, 3, 5, 6} and B = {2, 4} find A × B and B × A.

18. Let R = {(x, y) : x, y ∈ N and 2x + y = 9}, N being the set of all natural numbers. Write R as the set of ordered pairs.

19. Let R : A → B, where A = {3, 5} and B = {7, 11} and R = {(a, b) ∈ A × B | a – b is an odd number}.Write the

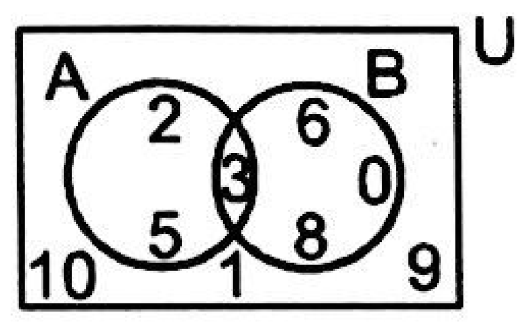
relation R.

Type (III) : Long Answer Type Questions : [04 Mark Each]

20. Write the set of all the possible subsets (power set) of the set :
(i) A = {a, b} (ii) B = {a, b, c}

21. If A = {a, b, c} ; B = {d}, C = {e} verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

22. From the adjoining Venn diagram, determine the following sets :



(i) $A \cup B$ (ii) $A \cap C$ (iii) $A - B$ (iv) $(A \cap B)'$

23. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find
(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

24. A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate information and if n(A) = n(B) then find the value of x.

25. If P = {a, b} find $P \times P \times P$.

26. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows : $(x, y) \in R \Leftrightarrow x$ divides y. Express R as a set of ordered pairs and determine the domain and range of R.

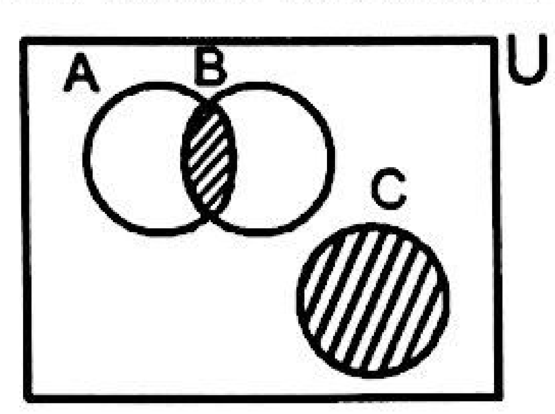
27. If R is relation 'is greater than' from A = {2, 3, 4, 5, 6} to B = {2, 5, 6} write the elements of R.

28. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Type (IV) : Very Long Answer Type Questions: [06 Mark Each]

29. Show that the relation R in the set (1, 2, 3) given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.

30. Express the shaded region of the adjoining Venn diagram in terms of union and intersection of the sets A, B and C :
Also if n(U) = 50, n(A) = 26
n(C) = 12, $n(A \cap B) = 20$ and
$n(A' \cap B') = 27$, find n(B) and $n(A' \cap B' \cap C')$



31. Let $U = \{x \in N : x \le 8\}$, $A = \{x \in N : 5 < x^2 < 50\}$ and $B = \{x \in N : x$ is prime number less than 10}. Draw a Venn diagram to show the relationship between the given sets. Hence list the elements of the following sets (i) A′ (ii) B′ (iii) A – B (iv) $A \cap B'$ (v) is $A - B = A \cap B'$?

32. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken:
(A) Only Chemistry (B) Only Mathematics (C) Only one of the subjects

33. A survey of 500 television viewers produced the given information : 285 watch football, 195 watch hockey, 115 watch cricket, 45 watch football and cricket, 70 watch football and hockey, 50 watch cricket and hockey, 50 do not watch any of the three games. How many watch exactly one of the three games ?

34. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. Find the number of students who have taken both Mathematics and Economics and the number of students who have taken Economics but not Mathematics, if it is given that each student has taken either Mathematics or Economics or both.

35. Let U = {1, 2, 3, 4, 5, 6, 7}, A = {2, 4, 6}, B = {3, 5} and C = {1, 2, 4, 7}. Find
(i) $A \cup B$ (ii) $B \cap C$ (iii) $A \cup (B \cap C)$ (iv) $(A \cap B) \cup C$ (v) $A' \cap B'$
(vi) $A \cap (B \cup C)'$ (vii) $A' \cup (B \cap C')$ (viii) A – B (ix) C – A (x) (B – A) $\cup$ (A – C)

36. If A = {1, 2, 3} and B = {a, b, c} then which of the following are relations from A to B ?
(i) R_1 = {(1, a), (1, b), (1, c)} (ii) R_2 = {(1, c), (2, b), (2, a)} (iii) R_3 = {(b, 2), (1, c), (3, a)}
(iv) R_4 = {(2, c), (3, c), (4, c)} (v) R_5 = {(1, d), (2, c), (3, a)}

Exercise # 1

SUBJECTIVE QUESTIONS

Section (A) : Representation of set, Types of sets, Subset, Power Set.

A-1. Write the set of all vowels in English alphabet which precede letter O.

A-2. Classify the following as a finite or infinite set :
(i) A = {x $\in$ N : (x – 1) (x – 2) = 0} (ii) B = {x $\in$ N : x is odd}

A-3. Write the following set by roster method : The set of all natural numbers 'x' such that 4x + 9 < 50.

A-4. Describe the following set by set property method {0, 3, 8, 15, 24, 35}

A-5. Describe the following set by roster method the set of all letters in the word TRIGONOMETRY.

A-6. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

A-7. Which of the following are true ?
(i) If A = {1, 5, 5, 5}, B = {1, 3, 5}, then $A \not\subset B$.
(ii) If A = {x : $x^3 - 1 = 0$, x $\in$ N}, B = {x : $x^2 - 4x + 3 = 0$, x $\in$ N} then $A \subseteq B$.

A-8. Assume that P(A) = P(B). Prove that A = B.

Section (B) : Venn diagrams, Algebra of sets.

B-1. If A = {x : x = 4n + 1, n $\le$ 5, n $\in$ N} and B {3n : n $\le$ 8, n $\in$ N}, then find A – (A – B).

B-2. Prove that $A \cup B = A \cap B$ iff A = B.

B-3. Prove that : A – (B $\cup$ C) = (A – B) $\cap$ (A – C) without using venn diagram.

B-4. Prove by using venn diagram
(i) A – (B $\cup$ C) = (A – B) $\cap$ (A – C) (ii) $A \subseteq B \Rightarrow B' \subseteq A'$

Section (C) : Theorems on cardinal number

C-1. A and B are two sets such that n(A) = 3 and n(B) = 6.
Find (i) minimum value of $n(A \cup B)$ (ii) maximum value of $n(A \cup B)$

C-2. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. Find the total number of members in the three athletic teams.

C-3. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Find the number of students who have taken exactly one subject.

Section (D) : Cartesian Product of Sets, Domain, Range and Co-domain of Relation.

D-1. Determine the domain and the range of the relation R defined by $R = \{(x + 1, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

D-2. If A = {3, 4, 6}, B = {1, 3} and C = {1, 2, 6} then find $(A - B) \times (A - C)$.

Section (E) : Types of Relations

E-1. Let n be a fixed positive integer. Define a relation R on the set of integers Z, $aRb \Leftrightarrow n|(a - b)$. Then prove that R is equivalence relation

E-2. Let R be a relation over the set $N \times N$ and it is defined by $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$. Then prove that R is equivalence relation

E-3. Let L be the set of all straight lines in the Euclidean plane. Two lines ℓ_1 and ℓ_2 are said to be related by the relation R if ℓ_1 is parallel to ℓ_2. Then prove that R is equivalence relation.

E-4. For $n, m \in N$, $n \mid m$ means that n is a factor of m, then prove that relation | is reflexive, transitive but not symmetric.

E-5. Let $R = \{(x, y) : x, y \in A, x + y = 5\}$ where A = {1, 2, 3, 4, 5} then prove that R is neither reflexive nor transitive but symmetric.

Exercise # 2

PART - I : OBJECTIVE QUESTIONS

*** Marked Questions may have more than one correct option.**

Section (A) : Representation of set, Types of sets, Subset, Power Set.

A-1. The set of intelligent students in a class is-
(A) a null set
(B) a singleton set
(C) a finite set
(D) not a well defined collection

A-2. Which of the following is the empty set
(A) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$
(B) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$
(C) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$
(D) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

A-3. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is

(A) Null set (B) Singleton set (C) Infinite set (D) None of these

A-4. If A = {x : |x| < 3, x ∈ Z} then the number of subsets of A is -
(A) 120 (B) 30 (C) 31 (D) 32

A-5. Which of the following are true ?
(A) $[3, 7] \subseteq (2, 10)$ (B) $(0, \infty) \subseteq (4, \infty)$ (C) $(5, 7] \subseteq [5, 7)$ (D) $[2, 7] \subseteq (2.9, 8)$

A-6. The number of subsets of the power set of set A = {7, 10, 11} is
(A) 32 (B) 16 (C) 64 (D) 256

Section (B) : Venn diagrams, Algebra of sets.

B-1. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?
(A) 3 (B) 6 (C) 9 (D) 18

B-2. Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then $A \cup (B \cap C)$ is
(A) {3} (B) {1, 2, 3, 4} (C) {1, 2, 4, 5} (D) {1, 2, 3, 4, 5, 6}

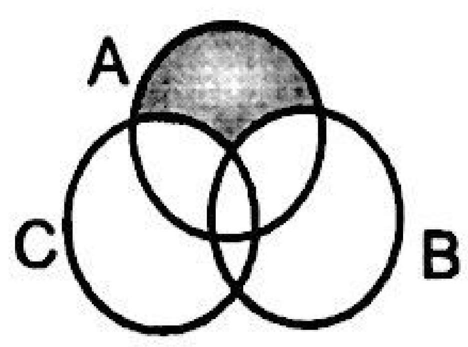
B-3. Let A = {x : x ∈ R, |x| < 1} , B = {x : x ∈ R, |x – 1| ≥ 1} and $A \cup B$ = R – D, then the set D is
(A) {x : 1 < x ≤ 2} (B) {x : 1 ≤ x < 2} (C) {x : 1 ≤ x ≤ 2} (D) None of these

B-4. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(A) {2, 3, 5} (B) {3, 5, 9} (C) {1, 2, 5, 9} (D) None of these

B-5. Let A and B be two sets. Then
(A) $A \cup B \le A \cap B$ (B) $A \cap B \le A \cup B$ (C) $A \cap B = A \cup B$ (D) None of these

B-6. If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then $(A \cap B) \cup (A \cap C)$ is equal to
(A) {3, 4, 10} (B) {2, 8, 10} (C) {4, 5, 6} (D) {3, 5, 14}

B-7. The shaded region in the given figure is



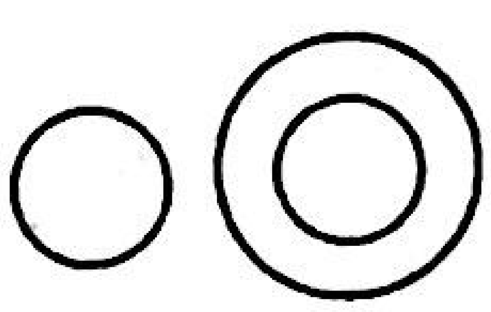
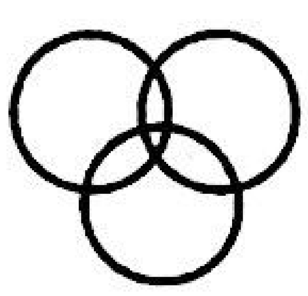
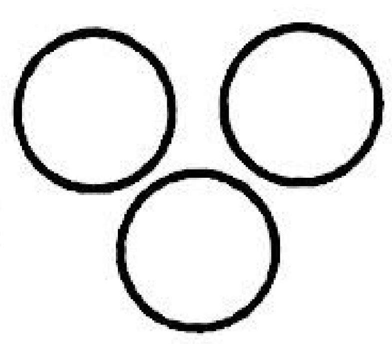
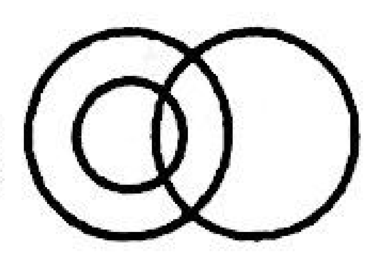
(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \cap (B - C)$ (D) $A - (B \cup C)$

B-8. Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then $n(A' \cap B') =$
(A) 400 (B) 600 (C) 300 (D) 200

B-9. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7}, then $A \cap B'$ is
(A) B′ (B) A (C) A′ (D) B

B-10. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) ; n \in N\}$, then $X \cup Y$ is equal to
(A) X (B) Y (C) N (D) None of these

B-11. Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(A)  (B)  (C)  (D) 

Section (C) : Theorems on cardinal number

C-1. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspaper is-
(A) at least 30 (B) at most 20 (C) exactly 25 (D) none of these

C-2. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then number of families which buy newspaper A only is
(A) 3100 (B) 3300 (C) 2900 (D) 1400

C-3. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is
(A) 80 percent (B) 40 percent (C) 60 percent (D) 70 percent

C-4. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?
(A) 35 (B) 48 (C) 60 (D) 22

Section (D) : Cartesian Product of sets, Domain, range and co-domain of Relation.

D-1. If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to
(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \times (B \cup C)$ (D) $A \times (B \cap C)$

D-2. If A = {2, 4, 5}, B = {7, 8, 9}, then $n(A \times B)$ is equal to
(A) 6 (B) 9 (C) 3 (D) 0

D-3. If $A = \{x : x^2 - 5x + 6 = 0\}$, B = {2, 4}, C = {4, 5} then $A \times (B \cap C)$ is-
(A) {(2, 4), (3, 4)} (B) {(4, 2), (4, 3)} (C) {(2, 4), (3, 4), (4, 4)} (D) {(2, 2), (3, 3), (4, 4), (5, 5)}

D-4. Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R can equal to set
(A) A (B) B (C) $A \times B$ (D) $B \times A$

D-5. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relation which can be defined from A to B is
(A) 2^5 (B) $2^{10} - 1$ (C) $2^{12} - 1$ (D) none of these

D-6. Let R be relation from a set A to a set B, then
(A) $R = A \cup B$ (B) $R = A \cap B$ (C) $R \subseteq A \times B$ (D) $R \subseteq B \times A$

D-7. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is not a relation from X to Y
(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ (B) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}
(C) R_3 = {(1, 1), (1, 3) (3, 5), (3, 7), (5, 7)} (D) R_4 = {(1, 3), (2, 5), (2, 4), (7, 9)}

Section (E) : Types of Relations

E-1. Let A = {1, 2, 3, 4} and R be a relation in A given by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}, then relation R is
(A) Reflexive (B) Symmetric (C) Equivalence (D) Reflexive and Symmetric

E-2. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is
(A) Reflexive but not symmetric (B) Reflexive but not transitive

(C) Symmetric and Transitive (D) Neither symmetric nor transitive

E-3. The relation "less than" in the set of natural number is
(A) Only symmetric (B) Only transitive (C) Only reflexive (D) Equivalence relation

E-4. The relation R defined in N as aRb $\Leftrightarrow$ b is divisible by a is
(A) Reflexive but not symmetric (B) Symmetric but not transitive
(C) Symmetric and transitive (D) None of these

E-5. In the set A = {1, 2, 3, 4, 5} a relation R is defined by R = {(x, y)| x, y $\in$ A and x < y}. Then R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-6. For real numbers x and y, we write x R y $\Rightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-7. Which one of the following relations on R is equivalence relation-
(A) $x R_1 y \Leftrightarrow |x| = |y|$ (B) $x R_2 y \Leftrightarrow x \geq y$
(C) $x R_3 y \Leftrightarrow x \mid y$ (x divides y) (D) $x R_4 y \Leftrightarrow x < y$

E-8. The relation R defined in A = {1, 2, 3} by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false-
(A) R = {(1, 2), (2, 2), (3, 3), (2, 1), (2, 3), (3, 2)} (B) Co-domain of R = {1, 2, 3}
(C) Domain of R = {1, 2, 3} (D) Range of R = {1, 2, 3}

E-9. Let P = {(x, y) | $x^2 + y^2 = 1$, x, y $\in$ R}, then P is
(A) reflexive (B) symmetric (C) transitive (D) equilance

E-10. Let A = {p, q , r}. Which of the following is an equivalence relation on A ?
(A) R_1 = {(p, q), (q, r), (p, r) (p, p)} (B) R_2 = {(r, q), (r, p), (r, r) (q, q)}
(C) R_3 = {(p, p), (q, q), (r, r) (p, q)} (D) none of these

E-11. Let R_1 be a relation defined by R_1 = {(a, b)| a $\geq$ b ; a, b $\in$ R} . Then R_1 is
(A) An equivalence relation on R (B) Reflexive, transitive but not symmetric
(C) Symmetric, Transitive but not reflexive (D) Neither transitive nor reflexive but symmetric

E-12. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. The R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-13. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is
(A) Reflexive and symmetric but not transitive (B) Reflexive, transitive but not symmetric
(C) Symmetric, transitive but not reflexive (D) Reflexive, transitive and symmetric

E-14. Let R be a relation on the set N be defined by {(x, y)| x, y $\in$ N, 2x + y = 41}. Then R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

PART - II : COMPREHENSION

Comprehension # 1 (Q. 1 to 3)

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Number of people who can speak Hindi only is
(A) 300 (B) 400 (C) 500 (D) 600

2. Number of people who can speak Bengali only is
(A) 150 (B) 250 (C) 50 (D) 100

3. Number of people who can speak both Hindi and Bengali is
(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2 (Q. 4 to 6)

Let R be a relation defined as $R = \{ (x, y) : y = |x - 1|, x \in Z \text{ and } |x| \le 3\}$

4. Relation R is equal to :
(A) {(1, 0), (1, 2), (3, 2), (4, 3)}
(B) {(–3, 4), (–2, 3), (–1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}
(C) {(4, –3), (3, –2), (2 –1), (1, 0), (2, 3)}
(D) None of these

5. Domain of R is :
(A) {0, 1, 2, 3, 4}
(B) {1, 3, 4}
(C) {– 3, – 2, – 1, 0, 1, 2, 3}
(D) {0, 1, 2, 3, 4}

6. Range of R is
(A) {0, 1, 2, 3, 4}
(B) {–3, –2, –1, 0, 1, 2, 3}
(C) {–4, –3, –1, –2, 0}
(D) {–1, 0, 1, 2, 3, 4}

Exercise # 3

PART - I : AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is-
(1) transitive (2) not symmetric (3) reflexive (4) a function **[AIEEE-2004]**

2. Let R = {(3, 3), (6, 6), (9, 9), (12, 12) (6, 12), (3, 9), (3, 12), (3, 6)} be relation on the set A = {3, 6, 9, 12}. Then the relation R is **[AIEEE - 2005]**
(1) reflexive and transitive only (2) reflexive only
(3) an equilvalence relation (4) reflexive and symmetric only

3. Let W denote the words in the english dictionary. Define the relation R by : $R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$. Then R is- **[AIEEE - 2006]**
(1) reflexive, symmetric and not transitive (2) reflexive, symmetric and transitive
(3) reflexive, not symmetric and transitive (4) not reflexive, symmetric and transitive

4. Let R be the real line. Consider the following subsets of the plane R × R **[AIEEE-2008]**

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$T = \{(x, y) : x - y \text{ is an integer}\}$

Which one of the following is true ?
(1) T is an equivalence relation on R but S is not
(2) Neither S nor T is an equivalence relation on R
(3) Both S and T are equivalence relations on R
(4) S is an equivalence relation on R but T is not

5. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009]**
(1) A = C (2) B = C (3) $A \cap B = \phi$ (4) A = B

6. Consider the following relations : **[AIEEE-2010]**
R : {(x, y)| x ,y are real numbers and x = wy for some rational number w}

$S = \{\left(\frac{m}{n}, \frac{p}{q}\right) \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$

Then
(1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation
(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not an equivalence relation

[AIEEE-2011, I]

7. Let R be the set of real numbers.
Statement-1 : $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

8. Consider the following relation R on the set of real square matrices of order 3. **[AIEEE - 2011, II]**
$R = \{(A, B) \mid A = P^{-1} BP$ for some invertible matrix $P\}$.
Statement -1 : R is equivalence relation.
Statement - 2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.
(1) Statement-1 is true, statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

9. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012]**
(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

10. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[AIEEE - 2013, (4, –¼),360]**
(1) 256 (2) 220 (3) 219 (4) 211

PART - II : CBSE PROBLEMS (PREVIOUS YEARS)

1. Show that the relation R defined by $(a, b) R(c, d) \Rightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation. **[CBSE - 2008]**

2. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$, is an equivalence relation. **[CBSE - 2009]**

3. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b \in Z$, and $(a - b)$ is divisible by 5$\}$ Prove that R is an equivalence relation. **[CBSE - 2010]**

4. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R$ and $a \leq b^3\}$ is neither reflexive, nor symmetric nor transitive. **[CBSE - 2010]**

5. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1. **[CBSE - 2010]**

6. Show that the relation S defined on the set $N \times N$ by $(a, b) S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation. **[CBSE - 2010]**

7. Let $f : X \rightarrow Y$ be a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is an equivalence relation on X **[CBSE - 2010]**

8. State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive. **[CBSE - 2011]**

Answers

BOARD LEVEL SOLUTIONS

1. Boys of class XI + Girls of class XI = Students of class XI
   $\therefore$ A′ = Not A = No. of boys in class XI of school S

2. $(A \cup B)' \cup (A \cap B)$ or $(A - B)' \cap (B - A)'$

3. $x - 1 = 2, -5 = y + 3$

4. B × A can be obtained from A × B by interchanging the entries in A × B.
   $\therefore$ B × A = {(1, a), (2, a), (5, a), (1, b), (2, b), (5, b)}

5. A is the set of first entries and B is the set of second entries
   $\therefore$ A = {a, b}, B = {1, 2, 3}

6. n(A) = n ,No. of elements in A × A = n^2

   Total relations defined for $A \to A = 2^{n^2}$

7. Number of elements in A × B = mn
   $\Rightarrow$ Number of relations from A to B = 2^{mn}

8. Let $(a, b) \in R$
   $\therefore\ a \le b \not\Rightarrow b \le a$
   $\therefore\ (b, a) \notin R$
   Hence R is not symmetric.

9. Since $(1, 1) \notin R$ also $(2, 2) \notin R$ and $(3, 3) \notin R$
   $\therefore\ (a, a) \notin R$
   Hence R is not reflexive.

10. $B \cup C = \{1, 2, 3, 5, 7, 8\}$
    $A - (B \cup C) = \{4\}$
    $A - B = \{2, 4\}$
    $A - C = \{1, 3, 4\}$
    $(A - B) \cap (A - C) = \{4\}$

11. (i) A′ = {0, 2, 4, 5, 6, 8}
    (ii) B′ = {1, 2, 3, 4, 5, 6, 7, 8, 9}
    (iii) C′ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
    (iv) D′ = {0, 1, 2, 4, 5, 6}

12. $A \cup B = \{1, 2, 3, 4, 5, 6\}$
    $(A \cup B)' = \{7, 8, 9, 10\}$
    $A' = \{1, 6, 7, 8, 9, 10\}$
    $B' = \{7, 8, 9, 10\}$
    $A' \cap B' = \{7, 8, 9, 10\}$

13. $B - C = \{e, o\}$
    $A \cap (B - C) = \{e\}$
    $A \cap B = \{e\}$
    $A \cap C = \{a, i\}$
    $(A \cap B) - (A \cap C) = \{e\}$

14. $A \cap B = \{2\}$
    $(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$
    $A' = \{1, 3, 5, 7, 9\}$
    $B' = \{1, 4, 6, 8, 9\}$
    $A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

15. n(U) = 40, n(T) = 26, n(C) = 18
    $n(T' \cap C') = 8 \Rightarrow n(T \cup C)' = 8$
    $\Rightarrow n(U) - n(T \cup C) = 8$
    $\Rightarrow n(T \cup C) = 32$
    $\Rightarrow n(T) + n(C) - n(T \cap C) = 32$
    $\Rightarrow n(T \cap C) = 12$

16. The subsets are ϕ, {–1}, {0}, {1}, {–1, 0}, {–1, 1}, {0, 1}, {–1, 0, 1}

17. A × B = {(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4), (6, 2), (6, 4)}
    B × A = {(2, 1), (2, 3), (2, 5), (2, 6), (4, 1), (4, 3), (4, 5), (5, 6)}

18. As $y = 9 - 2x$ so if $x = 1, y = 7$; $x = 2, y = 5$ etc.
    $\therefore$ R = {(1, 7), (2, 5), (3, 3), (4, 1)}

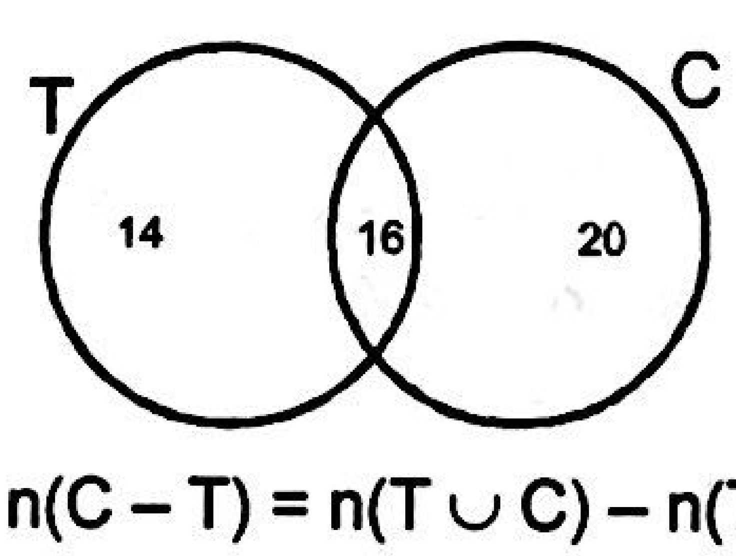
19. In set A and set B, elements are odd numbers so a – b i.e. difference of two odd number is always even.
    so relation R is empty relation.

20. (i) P (A) = {ϕ, {a}, {b}, {a, b}}
    (ii) B = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}}

21. $B \cup C = \{d, e\}$
    $A \times (B \cup C)$ = {a, b, c} × {d, e} = {(a, d), (a, e), (b, d), (b, e), (c, d), (c, e)}
    A × B = {a, b, c} × {d} = {(a, d), (b, d), (c, d)} ...(i)
    A × C = {a, b, c} × {e} = {(a, e), (b, e), (c, e)}
    $(A \times B) \cup (A \times C)$ = {(a, d), (b, d), (c, d), (a, e), (b, e), (c, e)} ...(ii)
    from (i) and (ii), L.H.S. = R.H.S.

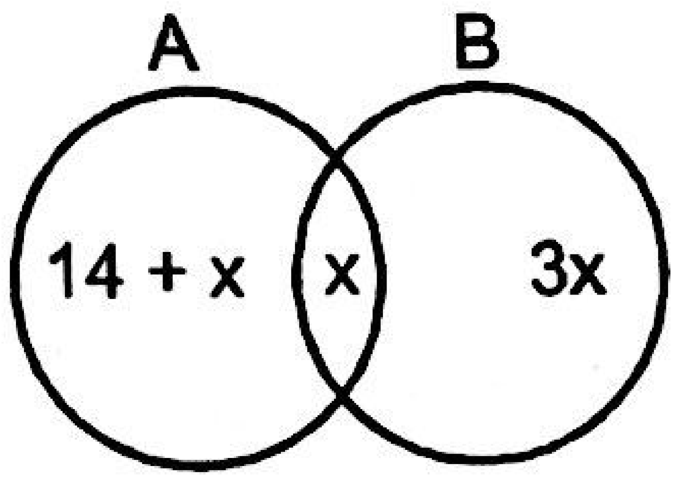
22. (i) $A \cup B = \{0, 2, 3, 5, 6, 8\}$
    (ii) $A \cap C = \{3\}$
    (iii) $A - B = \{2, 5\}$
    (iv) $(A \cap B)' = \{0, 1, 2, 5, 6, 8, 9, 10, 12\}$

23. T : people drinking tea
    C : people drinking coffee
    (i) $n(T) = n(T - C) + n(T \cap C)$
    $\Rightarrow 30 = 14 + n(T \cap C) \Rightarrow n(T \cap C) = 16$



    (ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

24. $n(A) = n(B)$
$\Rightarrow n(A - B) + n(A \cap B) = n(B - A) + n(A \cap B)$
$\Rightarrow 14 + x + x = 3x + x$
$\Rightarrow 14 = 2x \Rightarrow x = 7$



25. $P \times P \times P = \{a, b\} \times \{a, b\} \times \{a, b\} = \{(a, a), (a, b), (b, a), (b, b)\} \times \{a, b\}$

26. a|b stands for 'a divides b'. For the elements of the given sets A and B, we find that 2|6, 2|10, 3|3, 3|6 and 5|10
$\therefore (2, 6) \in R,, (2, 10) \in R, (3, 3) \in R, (3, 6) \in R$ and $(5, 10) \in R$
Thus, $R = \{(2, 6), (2, 10), (3, 3), (3, 6), (5, 10)\}$
Also Domain of $R = \{2, 3, 5\}$,
Range of $R = \{3, 6, 10\}$

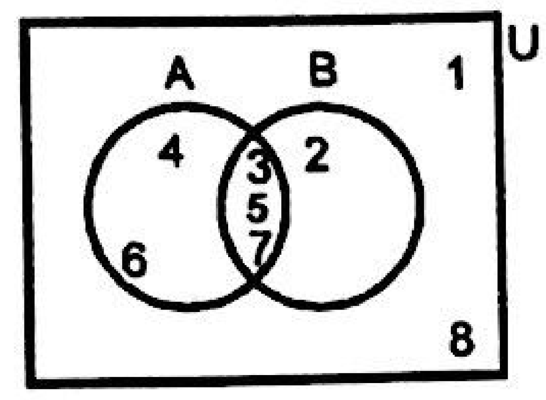
27. If $x \in A, y \in B$ then $x > y$,
so $R = \{(3, 2), (4, 2), (5, 2), (6, 2), (6, 5)\}$

28. Here $R = \{(a, b) : b = a + 1\}$
$= \{(a, a + 1) : a, a + 1 \in \{1, 2, 3, 4, 5, 6\}\}$
$= \{(1, 2)\ (2, 3), (3, 4), (4, 5), (5, 6)\}$
(i) R is not reflexive as $(a, a) \notin R \ \forall a$.
(ii) R is not symmetric as $(a, b) \in R$ but $(b, a) \notin R$
(iii) R is not transitive as $(a, b) \in R$ and $(a, c) \in R$ but $(a, c) \notin R$
$\because (1, 2) \in R, (2, 3) \in R$ but $(1, 3) \notin R$

29. Since (1, 1), (2, 2) and (3, 3) lie in R
$\therefore$ R is reflexive.
Also $(1, 2) \in R$ but $(2, 1) \notin R$
$\therefore$ R is not symmetric
since $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$
$\therefore$ R is not transitive.

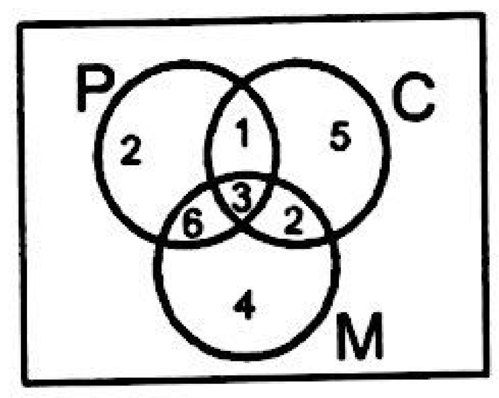
30. Shaded region is $(A \cap B) \cup C$
$n(A' \cap B') = 27 \Rightarrow n(A \cup B)' = 27$
$\Rightarrow n(U) - n(A \cup B) = 27$
$\Rightarrow n(A \cup B) = 50 - 27 = 23$
$\Rightarrow n(A) + n(B) - n(A \cap B) = 23$
$\Rightarrow n(B) = 23 - 26 + 20 = 17$
$n(A' \cap B' \cap C') = n(A \cup B \cup C)'$
$= n(U) - n(A \cup B \cup C) = 50 - 23 - 12 = 15$

31. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{3, 4, 5, 6, 7\}$; $B = \{2, 3, 5, 7\}$
$A' = \{1, 2, 8\}$
$B' = \{1, 4, 6, 8\}$



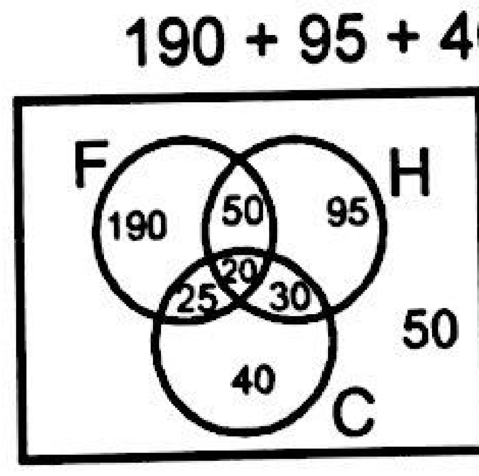
$A - B = \{x : x \in A \text{ and } x \notin B\} = \{4, 6\}$
$A \cap B' = \{3, 4, 5, 6, 7\} \cap \{1, 4, 6, 8\} = \{4, 6\}$

32. $n(P \cup C \cup M) = 25$
$n(M) = 15, n(P) = 12, n(C) = 11$
$n(M \cap C) = 5, n(M \cap P) = 9, n(P \cap C) = 4$

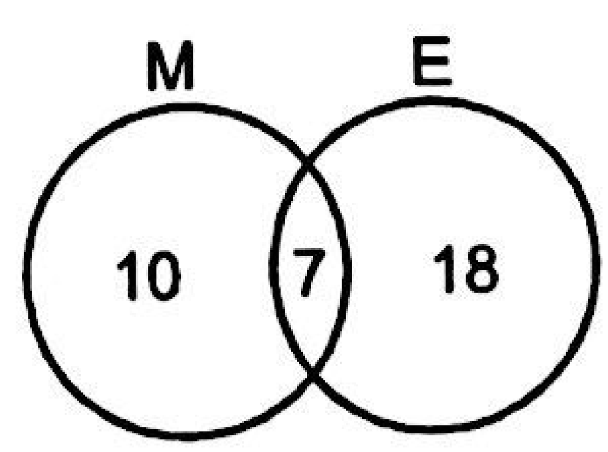


$n(P \cap C \cap M) = 3$
From Venn diagram n(only C) = 5,
n(only M) = 4, n(only P $\cup$ Only C $\cup$ Only M) = 11

33. $n(F \cup H \cup C) = n(U) - n(F \cup H \cup C)'$
$= 500 - 50 = 450$
$n(F \cup H \cup C) = n(F) + n(H) + n(C) - n(F \cap H) - n(H \cap C) - n(C \cap F) + n(F \cap C \cap H)$
$\Rightarrow 450 = 285 + 195 + 115 - 70 - 50 - 45 + n(F \cap C \cap H)$
$\Rightarrow n(F \cap C \cap H) = 20$
From Venn diagram required number of viewers =
$190 + 95 + 40 = 325$



34. M : Mathematics, E : Economics



$n(M \cup E) = 35, n(M) = 17, n(M - E) = 10$
$n(M \cap E) = n(M) - n(M - E) = 17 - 10 = 7$
$n(E - M) = n(E \cup M) - n(M) = 35 - 17 = 18$

35. (i) $A \cup B = \{2, 3, 4, 5, 6\}$ (ii) $B \cap C = \phi$
(iii) {2, 4, 6} (iv) {1, 2, 4, 7} (v) {1, 7}
(vi) {6} (vii) {1, 3, 5, 7} (viii) {2, 4, 6}
(ix) {1, 7} (x) {3, 5, 6}

36. (i) relation, as domain of $R_1 \subset A$.
(ii) relation, as domain of $R_2 \subset A$.
(iii) not a relation, as domain of $R_3 \not\subset A$.
(iv) not a relation, as domain of $R_4 \not\subset A$.
(v) not a relation, as range of $R_5 \not\subset B$.

EXERCISE # 1

Section (A) :

A-1. {a, e, i}

A-2. (i) Finite (ii) Infinite

A-3. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A-4. $\{x : x = n^2 - 1, n \in N, n \le 6\}$

A-5. {T, R, I, G, O, N, M, E, Y}

A-6. n = 3, m = 6 **A-7.** (i) False (ii) True

Section (B) :

B-1. {9, 21}

Section (C) :

C-1. (i) 6(ii) 9 **C-2.** 43 **C-3.** 22

Section (D) :

D-1. Domain = {1, 2, 3, 4, 5, 6},
Range = {5, 6, 7, 8, 9, 10}

D-2. {(4, 3), (4, 4), (6, 3), (6, 4)}

EXERCISE # 2

PART - I

Section (A) :

A-1. (D) **A-2.** (B) **A-3.** (A)

A-4. (D) **A-5.** (A) **A-6.** (D)

Section (B) :

B-1. (B) **B-2.** (B) **B-3.** (B)

B-4. (B) **B-5.** (B) **B-6.** (A)

B-7. (D) **B-8.** (C) **B-9.** (B)

B-10. (B) **B-11.** (D)

Section (C) :

C-1. (C) **C-2.** (B) **C-3.** (C)

C-4. (C)

Section (D) :

D-1. (C) **D-2.** (B) **D-3.** (A)

D-4. (C) **D-5.** (D) **D-6.** (C)

D-7. (D)

Section (E) :

E-1. (D) **E-2.** (A) **E-3.** (B)

E-4. (A) **E-5.** (C) **E-6.** (A)

E-7. (A) **E-8.** (A) **E-9.** (B)

E-10. (D) **E-11.** (B) **E-12.** (B)

E-13. (A) **E-14.** (D)

PART - II

1. (D) **2.** (B) **3.** (C) **4.** (B) **5.** (C) **6.** (A)

EXERCISE # 3

PART - I

1. (2) **2.** (1) **3.** (1) **4.** (1) **5.** (2) **6.** (2)

7. (3) **8.** (2) **9.** (2) **10.** (3)

PART - II

1. Reflexive
$\because a + b = b + a \Rightarrow (a, b) R(a, b)$
Hence R is reflexive.
(ii) Symmetric
$(a, b) R(c, d) \Rightarrow a + d = b + c$
$\Rightarrow b + c = a + d \Rightarrow c + b = d + a$
$\Rightarrow (c, d) R(a, b)$
Hence R is symmetric
(iii) $(a, b) R(c, d)$ and $(c, d) R(e, f)$
$\Rightarrow a + d = b + c$ and $c + f = d + e$
$\Rightarrow a + d + c + f = b + c + d + e$
$\Rightarrow a + f = b + e \Rightarrow (a, b) R(e, f)$
Hence R is transitive
Therefore, relation R is an equivalence relation.

2. **Reflexive :** Let $a \in A$
$\therefore |a - a| = 0$ is an even number
$\Rightarrow (a, a) \in R$
$\therefore$ R is reflexive
Symmetric : Let $a, b \in R$
Let $(a, b) \in R$
$\Rightarrow |a - b|$ is even $\Rightarrow |-(b - a)|$ is even
$\Rightarrow |b - a|$ is even
$\Rightarrow (b, a) \in R$
$\therefore$ R is symmetric
Transitive : Let $a, b, c \in R$
Let $(a, b) \in R$ and $(b , c) \in R$
$\Rightarrow |a - b|$ is even and $|b - c|$ is even
$\Rightarrow a - b$ is even and $b - c$ is even

$\Rightarrow$ (a – b) + (b – c) is even
$\Rightarrow$ |a – c| is even $\Rightarrow$ (a, c) $\in$ R
$\therefore$ R is transitive.
Hence R is an equivalence relation as R is reflexive, symmetric and transitive.

3. **Reflexive :** $\because$ a – a is divisible by 5 for all a $\in$ Z
$\therefore$ R is reflexive
Symmetric : (a, b) $\in$ R $\Rightarrow$ a – b is divisible by 5
$\Rightarrow$ b – a is divisible by 5
$\Rightarrow$ b – a $\in$ R
$\therefore$ R is symmetric
Transitive : (a, b) $\in$ R and (b, c) $\in$ R
$\Rightarrow$ a – b and b – c are both divisible by 5
$\Rightarrow$ a – b + b – c is divisible by 5
$\Rightarrow$ a – c is divisible by 5
$\Rightarrow$ (a, c) $\in$ R
$\therefore$ R is transitive
Since R is reflexive, symmetric and transitive.
Hence, R is an equivalence relation.

4. (i) As $a \leq a^3$ is not true for all a $\in$ R
$\therefore$ R is not reflexive
For example, if $a = \frac{1}{3}$ then $a > a^3$
i.e. $a \leq a^3$ is not true
(ii) If (a, b) $\in$ R then need not imply that (b, a) $\in$ R
$\therefore$ R is not symmetric
For example, if (1, 2) $\in$ R but (2, 1) $\notin$ R,
As $1 \leq 2^3$ but $2 \nleq 1^3$
(iii) If (a, b) $\in$ R and (b, c) $\in$ R, then need not imply that (a, c) $\in$ R
$\therefore$ R is not transitive
For example (100, 5) $\in$ R and (5, 2) $\in$ R but (100, 2) $\notin$ R
As $100 \leq 5^3$ and $5 \leq 2^3$ but $100 \geq 2^3$.

5. **Reflexive :** For all a $\in$ A
|a – a| = 0 is divisible by 4 $\Rightarrow$ (a, a) $\in$ S
$\therefore$ S is reflexive
Symmetric : Let a, b $\in$ A, (a , b) $\in$ S
$\Rightarrow$ |a – b| is divisible by 4
$\Rightarrow$ |b – a| is divisibe by 4 $\Rightarrow$ (b, a) $\in$ S
$\therefore$ S is symmetric
Transitive : Let a, b, c $\in$ A, (a, b) $\in$ S, (b, c) $\in$ S
$\Rightarrow$ |a – b| is divisible by 4 and |b – c| is divisible by 4
$\Rightarrow$ (a – b) and (b – c) is divisible by 4
$\Rightarrow$ (a – b) + (b – c) = a – c is divisible by 4
$\Rightarrow$ |a – c| is divisible by 4 $\Rightarrow$ (a, c) $\in$ S
$\therefore$ S is transitive
Since S is reflexive, symmetric and transitive.
Hence, S is an equivalence relation.
The set of all elements of A, related to 1 is {1, 5, 9}

6. Reflexive
$\because$ a + b = b + a $\Rightarrow$ (a, b) R(a, b)
Hence R is reflexive.
(ii) Symmetric
(a, b) R(c, d) $\Rightarrow$ a + d = b + c
$\Rightarrow$ b + c = a + d $\Rightarrow$ c + b = d + a
$\Rightarrow$ (c, d) R(a, b)
Hence R is symmetric
(iii) (a, b) R(c, d) and (c, d) R(e, f)
$\Rightarrow$ a + d = b + c and c + f = d + e
$\Rightarrow$ a + d + c + f = b + c + d + e
$\Rightarrow$ a + f = b + e $\Rightarrow$ (a, b) R(e, f)
Hence R is transitive
Therefore, relation R is an equivalence relation.

7. **Reflexive :** For every a $\in$ X, since f(a) = f(a)
$\therefore$ (a, a) $\in$ R $\quad\therefore$ R is reflexive
Symmetric : Let (a, b) $\in$ R for every a, b $\in$ X
$\Rightarrow$ f(a) = f(b) $\Rightarrow$ f(b) = f(a) $\Rightarrow$ (b, a) $\in$ R
$\therefore$ R is symmetric
Transitive : Let a, b, c $\in$ X. Let (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ f(a) = f(b) and f(b) = f(c)
$\Rightarrow$ f(a) = f(c) $\Rightarrow$ (a, c) $\in$ R
$\therefore$ R is transitive.
Since R is reflexive, symmetric and transitive.
Therefore R is an equivalence relation.

8. In the case of transitive relation
(a, b), (b, c) $\in$ R
$\Rightarrow$ (a, c) $\in$ R
Here (1, 2) and (2, 1) $\in$ R but (1, 1) $\notin$ R.
So, it is not transitive.

Advanced Level Problems

SUBJECTIVE QUESTIONS

1. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$ then find $n(A \cap B)$.

2. In a battle, 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. Find the minimum value of x.

3. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, find the value of x.

4. Prove that $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$.

5. Let B be a subset of A and let $P(A : B) = \{S : B \subseteq S \subseteq A\}$. If $A = \{a, b, c, d\}$ and $B = \{a, b\}$, then list all the elements of $P(A : B)$.

6. Prove that the relation $R = \{(x, y) : x^2 = xy\}$ is reflexive and transitive but not symmetric

7. Consider a relation R defined on set of square matrices A of order 2 satisfying $A^2 = I$. If A, B are two such matrices then relation R is defined as $(A, B) \in R \Rightarrow (AB)^T = A^TB^T$. Prove that relation R is reflexive and symmetric.

Answers

1. 4 **2.** 10 **3.** $x \in [39, 63]$ **5.** {a, b}, {a, b, c}, {a, b, d}, {a, b, c, d}