[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.4 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[4 × 3 = 12]**

Q.1 If $\frac{2}{\log_b x} = \frac{1}{\log_a x} + \frac{1}{\log_c x}$, where a, b, c, x. belong to $(1, \infty)$, then

(A) $2b = a + c$ (B) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ (C*) $b^2 = ac$ (D) $b^2 = acx$

Q.2 If $x^2 + y^2 = 14xy$ and $2\log\left(k(x+y)\right) = (\log x + \log y)$, then the value of k is

(A) $\frac{1}{16}$ (B*) $\frac{1}{4}$ (C) $2\log 2$ (D) $\frac{\log 14}{2}$

Q.3 Given that $\log_{10}(4252) = 3.6286$ then $\text{antilog}_{10}(0.6286)$ is

(A*) 4.252 (B) 2.52 (C) 3.62 (D) 6.286

Q.4 If $13^{\log_{10} x} = 338 - x^{\log_{10} 13}$, then the value of $(x+2)$ is equal to

(A) 13 (B) 15 (C) 17 (D*) 102

Q.5 to Q.8 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[4 × 4 = 16]**

Q.5 If $\log_a 4 = c, \log_b a = -1$ and $\log_{\frac{1}{2}} b = -1$ then $(4a^2 + b^2 + c^2)$ equals

(A) 7 (B) 8 (C*) 9 (D) 10

Q.6 Let $a = \frac{\log_{27} 8}{\log_3 2}$, $b = \left(\frac{1}{2^{\log_2 5}}\right)\left(\frac{1}{5^{\log_5(0.1)}}\right)$ and $c = \frac{\log_4 27}{\log_4 3}$, then the value of $(a + b \div c)$, is

(A) 1 (B) $\frac{4}{3}$ (C*) $\frac{5}{3}$ (D) $\frac{2}{3}$

Q.7 If $7\log_p\left(\frac{16}{15}\right) + 5\log_p\left(\frac{25}{24}\right) + 3\log_p\left(\frac{81}{80}\right) = 8$, then p^{16} equals

(A) 16 (B) 1 (C) 2 (D*) 4

Q.8 If $\log_{30}(3) = \alpha$ and $\log_{30}(5) = \beta$, then $\log_{30}(8)$ is equal to

(A) $3(1 + \alpha - \beta)$ (B) $3(1 + \alpha + \beta)$ (C) $3(\alpha + \beta)$ (D*) $3(1 - \alpha - \beta)$

[MULTIPLE CORRECT CHOICE TYPE]

Q.9 to Q.11 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[3×4=12]**

Q.9 The equation $|x - 10| \log_2(x-3) = 2(x-10)$ has

(A*) no prime solution (B*) only one natural solution

(C*) two rational solutions (D) no composite solution

Q.10 Which two of the following equations have the same solution

(A) $x^{\log_{\sqrt{x}}(x-2)} = 9$

(B) $\log_7(2^x - 1) + \log_7(2^x - 7) = 1$

(C*) $\log_4(x + 12) \cdot \log_x 2 = 1$

(D*) $\log_3\left(1+\log_3\left(2^x - 7\right)\right) = 1$

Q.11 Which of the following statement(s) is/are correct?

(A) $\log_{10}x^2 = 2\log_{10}x$

(B*) The value of $\frac{3^{\log_5 4}}{4^{\log_5 3}}$ is equal to 1.

(C*) $\log_2 5 + \log_5 2 > 2$

(D*) Number of positive integers that logarithm of whose reciprocals to the base 10 has the characterstic (-2) is 90.

PART-B

[MATRIX TYPE]

Q.1 has **four** statements (A, B, C, D) given in **Column-I** and **five** statements (P, Q, R, S, T) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.

[3+3+3+3=12]

Q.1 **Column-I** contains logarithmic equations and entries in **column-II** describes qualitatively the nature of their solution. (Take base of the logarithm as 10 where not mentioned.)

	Column-I		**Column-II**
(A)	The equation $x^{\log_3 x} = 9$ has	(P)	Only integral solution(s)
(B)	The equation $\frac{\log(35 - x^3)}{\log(5 - x)} = 3$, has	(Q)	only prime solutions
(C)	The equation $9^{\log_{1/3}(x+1)} = 5^{\log_{1/5}(2x^2+1)}$, has	(R)	only even integral solution
(D)	The equation $4^{(x^2+2)} - (9)2^{(x^2+2)} + 8 = 0$, has	(S)	only irrational solutions
		(T)	only rational solution(s)

[Ans. (A) S; (B) P, Q, T; (C) P, R, T; (D) P, T]

PART-C

[INTEGER TYPE]

Q.1 to Q.3 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**) **[3×5=15]**

Q.1 Let $x = 5^{(\log_5 2 + \log_5 3)}$.
If 'd' denotes the number of digits before decimal in x^{30} and 'c' denotes the number of naughts after decimal before a significant digit starts in x^{-20}, then find the value of $(d - c)$.
[Take $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$.] [Ans. 0009]

Q.2 Let $p = \frac{(\log_2 2000)(\log_5 2000) - 4(\log_5 2000)}{\log_2 2000}$. Find $p \in N$. [Ans. 3]

Q.3 If $x = \alpha$ satisfies the equation $\log_6(2^{x+3}) - \log_6(3^x - 2) = x$, then find the value of 3^α.
[Ans. 4]