MATHEMATICS

TARGET JEE(ADVANCED)

Revision Exercise (Quadratic eqn.)

QUESTION BANK

ON

QUADRATIC EQUATION

Instructions: Please revise class room notes of Sudhir Jain before solving Q.Bank

Time Limit: 5 Sitting Each of 45 Minutes duration approx.

[STRAIGHT OBJECTIVE TYPE]

Q.1 The values of a for which the equation $\sqrt{a}\ \sin x - 2\cos x = \sqrt{2} + \sqrt{2-a}$ has solutions are

(A) $a > 0$ (B) $a \le 3$ (C) $0 \le a \le 2$ (D) $\sqrt{5} - 1 \le a \le 2$

Q.2 Let a and b be two distinct roots of the equation $x^3 + 3x^2 - 1 = 0$. The equation which has (ab) as its root is equal to

(A) $x^3 - 3x - 1 = 0$ (B) $x^3 - 3x^2 + 1 = 0$
(C) $x^3 + x^2 - 3x + 1 = 0$ (D) $x^3 + x^2 + 3x - 1 = 0$

Q.3 Let sin x and sin y be roots of the quadratic equation $a\sin^2\theta + b\sin\theta + c = 0$ ($a, b, c \in R$ and $a \ne 0$) such that $\sin x + 2\sin y = 1$, then the value of $(a^2 + 2b^2 + 3ab + ac)$ equals

(A) 0 (B) 1 (C) 2 (D) 4

Q.4 If two roots of the equation $(x-1)(2x^2 - 3x + 4) = 0$ coincide with roots of the equation $x^3 + (a+1)x^2 + (a+b)x + b = 0$ where $a, b \in R$ then $2(a+b)$ equals

(A) 4 (B) 2 (C) 1 (D) 0

Q.5 Let k be a real number such that $k \ne 0$. If α and β are non zero complex numbers satisfying $\alpha + \beta = -2k$ and $\alpha^2 + \beta^2 = 4k^2 - 2k$, then a quadratic equation having $\frac{\alpha+\beta}{\alpha}$ and $\frac{\alpha+\beta}{\beta}$ as its roots is equal to

(A) $4x^2 - 4kx + k = 0$ (B) $x^2 - 4kx + 4k = 0$ (C) $4kx^2 - 4x + k = 0$ (D) $4kx^2 - 4kx + 1 = 0$

Q.6 If x and y satisfy the relation $(x-1)^2 + y^2 = 1$, then the possible value of $(x+y)$ is equal to

(A) $\frac{-3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) $\frac{-1}{4}$

Q.7 Let $P(x) = x^2 + \frac{4x}{3} + \log_{10}(4.\bar{9})$, $A = \prod_{i=1}^{12} P(a_i)$ where $a_1, a_2, \ldots, a_{12}$ are positive reals and $B = \prod_{j=1}^{13} P(b_j)$ where $b_1, b_2, \ldots, b_{13}$ are non-positive reals, then which one of the following is always correct?

(A) $A > 0, B > 0$ (B) $A > 0, B < 0$ (C) $A < 0, B > 0$ (D) $A < 0, B < 0$

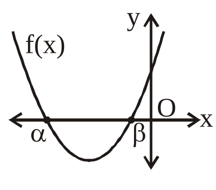
Q.8 The set of all real values of x for which both $\log_{\frac{x-2}{x+3}}(x^2 + x + 1)$ and $\sqrt{x^2 - 9}$ are meaningless, is equal to

(A) $[-4, -3]$ (B) $(-3, -2)$ (C) $(-3, 2]$ (D) $(-3, 1)$

Q.9 Let a_1 and a_2 be two values of a for which the expression $f(x, y) = 2x^2 + 3xy + y^2 + ay + 3x + 1$ can be factorised into two linear factors then the product (a_1a_2) is equal to

(A) 1 (B) 3 (C) 5 (D) 7

Q.10 The following figure shows the graph of $f(x) = ax^2 - bx + c$.
Then which one of the following is correct?



(A) $\frac{b}{c} > 0$ (B) a and c are of opposite sign
(C) a and b are of same sign (D) None

Q.11 If α, β, γ are the roots of the cubic $2010x^3 + 4x^2 + 1 = 0$, then the value of $(\alpha^{-2} + \beta^{-2} + \gamma^{-2})$ is equal to

(A) 8 (B) – 8 (C) 4 (D) – 4

Q.12 If exactly one root of the quadratic equation $x^2 - \left(k + \frac{11}{3}\right)x - (k^2 + k + 1) = 0$ lies in (0, 3) then which one of the following relation is correct?

(A) $-8 < k < -4$ (B) $-3 < k < -1$ (C) $1 < k < 4$ (D) $6 < k < 10$

Q.13 Let a, b and c be three distinct real roots of the cubic $x^3 + 2x^2 - 4x - 4 = 0$.

If the equation $x^3 + qx^2 + rx + s = 0$ has roots $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$, then the value of $(q + r + s)$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Q.14 Number of ordered pairs (x, y) of real numbers satisfying the equation $x^2 + y^2 - 24x - 26y + 313 = 0$ is equal to

(A) infinite (B) finite but more than one

(C) exactly one (D) zero

Q.15 If the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $\left(\frac{a}{a+b+c}\right)^2$ equals

(A) k^2 (B) $(k + 1)^2$ (C) $(k + 2)^2$ (D) $k^2 (k + 1)^2$

Q.16 If $c^2 = 4d$ and the two equations $x^2 - ax + b = 0$ and $x^2 - cx + d = 0$ have one common root, then the value of $2(b + d)$ is equal to

(A) $\frac{a}{c}$ (B) ac (C) 2ac (D) a + c

Q.17 If min. $(2x^2 - ax + 2) >$ max. $(b - 1 + 2x - x^2)$ then roots of the equation $2x^2 + ax + (2 - b) = 0$, are

(A) positive and distinct (B) negative and distinct

(C) opposite in sign (D) imaginary

Q.18 The number of integral values of α for which the inequality $x^2 - 2(4\alpha - 1)x + 15\alpha^2 > 2\alpha + 7$ is true for every $x \in R$, is

(A) 0 (B) 1 (C) 2 (D) 3

Q.19 If roots of the quadratic equation $bx^2 - 2ax + a = 0$ are real and distinct, where $a, b \in R$ and $b \neq 0$, then

(A) atleast one root lies in the interval (0, 1). (B) no root lies in the interval (0, 1).

(C) atleast one root lies in the interval (– 1, 0). (D) none of the above.

Q.20 Let $a, b, c \in R_0$ and 1 be a root of the equation $ax^2 + bx + c = 0$, then the equation $4ax^2 + 3bx + 2c = 0$ has

(A) imaginary roots (B) real and equal roots

(C) real and unequal roots (D) rational roots

Q.21 If p and q are the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha = 1$ $(\alpha \in R)$, then the minimum value of $(p^2 + q^2)$ is equal to

(A) 2 (B) 3 (C) 5 (D) 6

Q.22 Number of integral values of a for which every solution of the inequality $x^2 - 3x + 4 > 0$ is also the solution of the inequality $(a-1)x^2 - (a + |a-1| + 2)x + 1 \geq 0$, is

(A) 0 (B) 1 (C) 2 (D) 3

Q.23 If α and β are the roots of equation $x^2 - a(x+1) - b = 0$ where $a, b \in R - \{0\}$ and $a + b \neq 0$ then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} - \frac{2}{a+b}$ is equal to

(A) $\frac{4}{a+b}$ (B) $\frac{2}{a+b}$ (C) 0 (D) $\frac{1}{a+b}$

[COMPREHENSION TYPE]

Paragraph for question nos. 24 & 25

For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \; \forall x \in R$.
Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

Q.24 The value of $(a + b)$ is equal to

(A) 4 (B) 5 (C) 6 (D) 7

Q.25 The minimum value of $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to

(A) $\frac{-33}{8}$ (B) 0 (C) 4 (D) -2

Paragraph for question nos. 26 to 28

Consider a rational function $f(x) = \frac{x^2 - 3x - 4}{x^2 - 3x + 4}$ and a quadratic function
$g(x) = x^2 - (b+1)x + b - 1$, where b is a parameter.

Q.26 The sum of integers in the range of $f(x)$, is

(A) -5 (B) -6 (C) -9 (D) -10

Q.27 If both roots of the equation $g(x) = 0$ are greater than -1, then b lies in the interval

(A) $(-\infty, -2)$ (B) $\left(-\infty, \frac{-1}{4}\right)$ (C) $(-2, \infty)$ (D) $\left(\frac{-1}{2}, \infty\right)$

Q.28 The largest natural number b satisfying $g(x) > -2 \; \forall \; x \in R$, is

(A) 1 (B) 2 (C) 3 (D) 4

Paragraph for question nos. 29 to 31

Consider a function $f(x) = \frac{3x + a}{x^2 + 3}$ which has greatest value equal to $\frac{3}{2}$.

Q.29 The value of the constant number a is equal to

(A) 1 (B) 2 (C) 3 (D) 4

Q.30 The minimum value of $f(x)$ is equal to

(A) $\tan\left(\frac{-\pi}{3}\right)$ (B) $\sin\left(\frac{-\pi}{6}\right)$ (C) $\cos\left(\frac{-\pi}{3}\right)$ (D) $\cot\left(\frac{\pi}{2}\right)$

Q.31 If the equation $f(x) = b$ has two distinct real roots then the number of integral values of b is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Paragraph for question nos. 32 to 34

Consider two quadratic trinomials $f(x) = x^2 - 2ax + a^2 - 1$ and
$g(x) = (4b - b^2 - 5)\,x^2 - (2b - 1)\,x + 3b$, where $a, b \in R$.

Q.32 The values of a for which both roots of the equation $f(x) = 0$ are greater than -2 but less than 4, lie in the interval
(A) $-\infty < a < -3$ (B) $-2 < a < 0$ (C) $-1 < a < 3$ (D) $5 < a < \infty$

Q.33 If roots of the quadratic equation $g(x) = 0$ lie on either side of unity, then number of integral values of b is equal to
(A) 1 (B) 2 (C) 3 (D) 4

Q.34 If $f(x) < 0\ \forall\ x \in [0, 1]$, then a lie in the interval
(A) $-1 < a < 1$ (B) $0 < a < 2$ (C) $0 < a < 1$ (D) $a > 3$

[REASONING TYPE]

Q.35 **Statement-1:** The equation $(x - p)(x - r) + \sin\theta\,(x - q)(x - s) = 0$, where $p < q < r < s$ and $\theta \in R$ has non-real roots.
Statement-2 : If the equation $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a \neq 0$ has non-real roots then $b^2 - 4ac < 0$.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.36 **Statement-1:** Number of integral values of m for which exactly one root of the equation $x^2 - 2mx + m^2 - 1 = 0$ lies in the interval $(-2, 4)$ equals 2.
Statement-2: Let $f(x) = ax^2 + bx + c$ where $a, b, c \in R$ and $a \neq 0$. If $f(d)\,f(e) < 0$ then the equation $f(x) = 0$ has exactly one root in (d, e).
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.37 **Statement 1:** If $0 < \theta < \frac{\pi}{4}$, then the equation $(x - \sin\theta)(x - \cos\theta) - 2 = 0$ has both roots in the interval $(\sin\theta, \cos\theta)$.
Statement 2: Let $f(x) = px^2 + qx + r$ ($p, q, r \in R$ and $p \neq 0$) be such that $f(a)\,f(b) < 0$ then there exist exactly one solution of the equation $f(x) = 0$ in interval (a, b).
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

Q.38 **Statement-1:** If the equations $ax^2 + bx + c = 0$ ($a, b, c \in R$ and $a \neq 0$) and $2x^2 + 7x + 10 = 0$ have a common root, then $\frac{2a + c}{b} = 2$.
Statement-2: If both roots of $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are same, then
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. Given $a_1, b_1, c_1, a_2, b_2, c_2 \in R$ and $a_1a_2 \neq 0$.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false. (D) Statement-1 is false, statement-2 is true.

[MULTIPLE OBJECTIVE TYPE]

Paragraph for question nos. 39 to 41

Consider the expression $g(x) = \sin^2 x - (b+1)\sin x + 3(b-2)$ where b is a real parameter.

Q.39 Number of integral values of b for which the equation $g(x) = 0$ has exactly one root in the interval $[0, \pi]$ are

(A) 0 (B) 1 (C) 2 (D) 3

Q.40 If the equation $g(x) = 0$ have two distinct roots in $(0, \pi)$ then b lie in the interval

(A) (0, 3) (B) (1, 3) (C) (2, 3) (D) (0, 2)

Q.41 If $g(x)$ is non-negative for all real x, then b lie in the interval

(A) $[1, \infty)$ (B) $(-\infty, 1]$ (C) $[-1, 1]$ (D) $[3, \infty)$

Q.42 For $x \in R$, the expression $\dfrac{x^2+34x-71}{x^2+2x-7}$ can not lie between,

(A) (5, 7) (B) (12, 19) (C) (1, 4) (D) (8, 9)

Q.43 In which of the following inequalities, the set of all real values of x is same as the set of all real values of k for which the equation $kx^2 - 4x + k = 0$ has real roots and satisfying $1 - k \le 0$?

(A) $0 \le \log_2 x \le 1$ (B) $x^2 - 3x + 2 \le 0$

(C) $\sin(\pi x) \le 0$ in $[0, 2]$ (D) $|x - 1| \le 1$

Q.44 If the vertex of the parabola $y = 3x^2 - 12x + 9$ is (a, b), then the parabola whose vertex is (b, a), is(are)

(A) $y = x^2 + 6x + 11$ (B) $y = x^2 - 7x + 3$

(C) $y = -2x^2 - 12x - 16$ (D) $y = -2x^2 + 16x - 13$

Q.45 Let x and y be 2 real numbers which satisfy the equations

$(\tan^2 x - \sec^2 y) = \dfrac{5a}{6} - 3$ and $(-\sec^2 x + \tan^2 y) = a^2$, then the value of a can be equal to

(A) $\dfrac{2}{3}$ (B) $\dfrac{-2}{3}$ (C) $\dfrac{3}{2}$ (D) $\dfrac{-3}{2}$

Q.46 If the quadratic polynomial $P(x) = (p-3)x^2 - 2px + 3p - 6$ ranges from $[0, \infty)$ for every $x \in R$, then the value of p can be

(A) $\dfrac{3}{2}$ (B) 4 (C) 6 (D) 7

Q.47 Let a, b and c be real numbers. Which of the following statement(s) about the equation $(x-a)(x-b) = c$ is/are incorrect?

(A) If $c > 0$, then roots are always real. (B) If $c > 0$, then roots are always non-real.

(C) If $c < 0$, then roots are always real. (D) If $c < 0$, then roots are always non-real.

Q.48 If quadratic equation $x^2 + 2(a+2b)x + (2a+b-1) = 0$ has unequal real roots for all $b \in R$ then the possible values of a can be equal to

(A) 5 (B) – 1 (C) – 10 (D) 3

Q.49 Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$ be two quadratic polynomials with real coefficients and satisfy $ac = 2(b+d)$. Then which of the following is(are) correct?

(A) Exactly one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(B) Atleast one of either $f(x) = 0$ or $g(x) = 0$ must have real roots.

(C) Both $f(x) = 0$ and $g(x) = 0$ must have real roots.

(D) Both $f(x) = 0$ and $g(x) = 0$ must have imaginary roots.

Q.50 If all values of x which satisfies the inequality $\log_{\frac{1}{3}}\left(x^2+2px+p^2+1\right)\ge 0$ also satisfy the inequality $kx^2+kx-k^2\le 0$ for all real values of k, then all possible values of p lies in the interval

(A) [– 1, 1] (B) [0, 1] (C) [0, 2] (D) [– 2, 0]

Q.51 If a, b, c are sides of ΔABC and $a > b > c$, then the equation
$a(x-b)(x+c)+b(x-a)(x+c)-c(x-a)(x-b)=0$ has

(A) real and unequal roots (B) roots with opposite sign
(C) exactly one root in (b, a) (D) imaginary roots

[MATCH THE COLUMN]

Q.52 The expression $y=ax^2+bx+c$ ($a, b, c \in R$ and $a\ne 0$) represents a parabola which cuts the x-axis at the points which are roots of the equation $ax^2+bx+c=0$. Column-II contains values which correspond to the nature of roots mentioned in column-I.

	Column-I		**Column-II**
(A)	For $a=1, c=4$, if both roots are greater than 2 then b can be equal to	(P)	4
(B)	For $a=-1, b=5$, if roots lie on either side of -1 then c can be equal to	(Q)	8
(C)	For $b=6, c=1$, if one root is less than -1 and the other root greater than $\frac{-1}{2}$ then a can be equal to	(R)	10
		(S)	no real value

Q.53

	Column-I		**Column-II**
(A)	If $\alpha, \beta \in (0, \pi)$ and $\alpha\ne\beta$ satisfy the equation $\frac{1-\cos 2\theta}{\sin\theta}=\frac{1}{2}$, then the value of $\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)$ is equal to	(P)	0
		(Q)	8
(B)	If the expression $\frac{x^2+(2m+3)x+(m^2+3)}{\sqrt{x^2+(2m+1)x+m^2+2}}$ is non-negative $\forall\ x\in R$, then the possible values of m can be equal to	(R)	1
		(S)	– 1
(C)	If the parabola $y=5x^2+x-3$ lies above the parabola $y=2x^2+6x-1$, then integral values of x can be equal to	(T)	2
(D)	The number of real solutions of the equation $x^{2\log_x(x+3)}=16$ is equal to		

[SUBJECTIVE]

Q.54 Let M be the minimum value of $f(\theta)=(3\cos^2\theta+\sin^2\theta)(\sec^2\theta+3\,\text{cosec}^2\theta)$, for permissible real values of θ and P denotes the product of all real solutions of the equation $\frac{(x-1)(50-10x)}{x^2-5x}=x^2-8x+7$. Find (P M).

Q.55 If the range of values of a for which the roots of the equation $x^2-2x-a^2+1=0$ lie between the roots of the equation $x^2-2(a+1)x+a(a-1)=0$ is (p, q), find the value of $\left(q+\frac{1}{p^2}\right)$.

Q.56 Let x_1 and x_2 be the real roots of the equation $x^2-kx+(k^2+7k+15)=0$.
What is the maximum value of $(x_1{}^2+x_2{}^2)$?

Q.57 If sum of maximum and minimum value of $y = \log_2(x^4 + x^2 + 1) - \log_2(x^4 + x^3 + 2x^2 + x + 1)$ can be expressed in form $((\log_2 m) - n)$, where m and 2 are coprime then compute $(m + n)$.

Q.58 If $1 - \log_x 2 + \log_{x^2} 9 - \log_{x^3} 64 < 0$, then range of x is (a, b). Find the minimum value of (a + 9b).

Q.59 Let $f(x) = x^2 + ax + b$. If $\forall\, x \in R$, there exist a real value of y such that $f(y) = f(x) + y$, then find the maximum value of 100a.

Q.60 If α, β are roots of the equation $2x^2 + 6x + b = 0$ where $b < 0$, then find the least integral value of $\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)$.

Q.61 If all the solutions of the inequality $x^2 - 6ax + 5a^2 \le 0$ are also the solutions of inequality $x^2 - 14x + 40 \le 0$ then find the number of possible integral values of a.

Q.62 Find number of integral values of x satisfying $\log_4(3x^2 - 8x + 7) - \log_2(x - 2) \ge -\cot\frac{3\pi}{4}$.

Q.63 Find the number of integral values of a so that the inequation $x^2 - 2(a + 1)x + 3(a - 3)(a + 1) < 0$ is satisfied by atleast one $x \in R^+$.

Q.64 Suppose that a, b, c, d are rationals which satisfy $a + b + c + d = 10$, $(a + b)(c + d) = 16$, $(a + c)(b + d) = 21$ and $(a + d)(b + c) = 24$, then find the value of $(a^2 + b^2 + c^2 + d^2)$.

ANSWER KEY

Q.1	D	Q.2	A	Q.3	A	Q.4	C	Q.5	B
Q.6	D	Q.7	A	Q.8	C	Q.9	C	Q.10	D
Q.11	B	Q.12	B	Q.13	C	Q.14	C	Q.15	D
Q.16	B	Q.17	D	Q.18	B	Q.19	A	Q.20	C
Q.21	C	Q.22	A	Q.23	C	Q.24	B	Q.25	D
Q.26	B	Q.27	D	Q.28	B	Q.29	C	Q.30	B
Q.31	B	Q.32	C	Q.33	B	Q.34	C	Q.35	D
Q.36	D	Q.37	D	Q.38	A	Q.39	B	Q.40	ABC
Q.41	AD	Q.42	AD	Q.43	AB	Q.44	AC	Q.45	AD
Q.46	C	Q.47	BCD	Q.48	BC	Q.49	B	Q.50	ABC
Q.51	ABC	Q.52	(A) S (B) Q, R (C) P			Q.53	(A) Q; (B) P, S; (C) Q, S (D) P		
Q.54	0024	Q.55	0017	Q.56	0018	Q.57	0005	Q.58	0025
Q.59	0050	Q.60	0010	Q.61	0000	Q.62	0004	Q.63	0005
Q.64	0039								